# A modified rotating liquid optics method to measure the 

## gravity acceleration

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#### Abstract

The gravity acceleration is an important physics constant, which is measured by the rotating liquid as a university physical experiment. However, the present experimental methods have a bigger amount of experimental error. A modified rotating liquid optics method is presented to measure the gravity acceleration, in which an angle measurement instrument with an accuracy of 0.05 degree is introduced. Laser is used to measure the angles of the highest point and the lowest point of the rotating liquid. Then the gravity acceleration can be calculated. The modified method has more little error and makes the experiment easier and more convenient.


Keywords: Gravity acceleration, rotating liquid, angle measurement

## 1. INTRODUCTION

The gravity acceleration is an important physical constant, which varies with latitude and altitude. The measurement of gravity acceleration has great values both on theory and practice. At present, the gravity acceleration is measured by means of rotating liquid experiment device in the university physics experiment[1], in which the key step is to accurately measure the difference in height between the highest point and the lowest point of the rotating liquid[2]. However, this measurement method may cause great errors[3]. A modified rotating liquid optics method is presented to accurately measure the gravity acceleration, in which an angle measurement instrument with an accuracy of 0.05 degree is introduced. The measurement of distance between the highest point and the lowest point of the rotating liquid is converted to the angular measurement of the highest point and the lowest point of the rotating liquid by use of a small Laser. And the modified formula is deduced. Then the gravity acceleration can be calculated. The modified method has more little error and makes the experiment easier and more convenient.

## 2. THEORETICAL METHOD

The liquid is the water in the rotating liquid experiment. The rotating water is in a cylinder with a radius of $R$, and the angular speed of a rotation is denoted $\omega$, showed in Fig 1. According to the analysis of the water on the surface of the liquid, we can get

$$
\begin{equation*}
N \cos \theta=m g, N \sin \theta=m x \omega^{2} \tag{1}
\end{equation*}
$$

The liquid surface curve formula can be expressed as

$$
\begin{equation*}
\mathbf{y}=\frac{\omega^{2} x^{2}}{2 g}+y_{0} \quad 0 \leq x \leq R \tag{2}
\end{equation*}
$$

The point at $x=0$ is the lowest point of liquid, and the point at $x=R$ is the highest point. The height difference between the highest point and the lowest point is denoted by $\Delta h$. The gravity acceleration can be expressed as[4]
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$$
\begin{equation*}
\mathbf{g}=\frac{\omega^{2} R^{2}}{2 \Delta \mathbf{h}}=\frac{\pi^{2} D^{2}}{2 T^{2} \Delta \mathbf{h}} \tag{3}
\end{equation*}
$$

The cylinder diameter $D$, rotation period of the cylinder $T$ and the height difference of liquid $\Delta h$ can be measured by the equipment. Then the measurements are calculated with Formula (3), thus the value of the gravity acceleration can be calculated.

The key step in the measurement is to measure $\Delta h$, the height difference of liquid. The accuracy of the height difference determines the accuracy of the experimental results.


Figure 1. The surface curve of the rotating liquid
In the traditional method, the height difference of liquid $\Delta h$ is measured with the scale paper on the side wall of the cylinder, which just reaches millimeter scale accuracy. Meanwhile, the fluctuant liquid surface results in the reading errors caused by human factors.
In order to reduce those errors, an angle measuring instrument and a small laser are introduced to measure the height difference of liquid $\Delta h$. The angle measuring instrument has two arms, the fixed arm and the measuring arm. The fixed arm of the angle measuring instrument is horizontally arranged at a vertically bar with a certain height up the liquid. A laser is fixed on the end of the measuring arm of angle measuring instrument, which is in the direction of the measuring arm of angle measuring instrument. The lowest point of rotating liquid is the center point of the liquid, when the distance from the fixed arm of angle measuring instrument to the lowest point is denoted by $h_{I}$. The highest point of rotating liquid locates on the boundary between the liquid surface and the inner wall of the cylinder, when the distance from the fixed arm of angle measuring instrument to the highest point is denoted by $h_{2}$.

When laser spot is located precisely in the lowest point of liquid, the angle displayed on the angle measuring instrument is denoted by $\theta_{1}$. When laser spot is located precisely in the highest point of liquid, the angle displayed on the angle measuring instrument is denoted by $\theta_{2}$. Thus the height difference of liquid $\Delta h$ can be expressed by

$$
\begin{equation*}
\Delta h=h_{1}-h_{2}=(L-R) \tan \theta_{1}-(L-R-l) \tan \theta_{2} \tag{4}
\end{equation*}
$$

Then the gravity acceleration can be expressed as

$$
\begin{equation*}
\mathbf{g}=\frac{\omega^{2} R^{2}}{2 \Delta h}=\frac{\mathbf{2} \pi^{2} R^{2}}{T^{2}\left((L-R) \tan \theta_{1}-(L-R-l) \tan \theta_{2}\right)} \tag{5}
\end{equation*}
$$

Where $R$ is the radius of cylinder, $L$ is the distance from the angle measuring instrument to the vertically bar. $l$ is the shortest distance from the cylinder to the vertically bar. $T$ is the rotation period.


Figure 2. The principle of the modified measuring method

## 3. RESULTS AND DISCUSSION

According to the modified method, modified experiment equipment is designed and completed. Based on this equipment, the gravity acceleration is measured.
The gravity acceleration at Jinan, China is $9.7988 \mathrm{~m} / \mathrm{s}^{2}$. The gravity accelerations measured by traditional rotation liquid method are listed in Table 1. The same experiment is repeated six times, and the average value of six experiments is taken as the experiment value. From table 1, we can get that the gravity acceleration measured by the traditional method is $10.51 \mathrm{~m} / \mathrm{s}^{2}$. There is a big error between the experimental value and the accurate value $9.7988 \mathrm{~m} / \mathrm{s}^{2}$, and the relative error is $7.3 \%$.

Table 1. Dates by traditional method

| Nmber | Period <br> $(\mathrm{T} / \mathrm{S})$ | $\Delta H(\mathrm{~cm})$ | $\mathrm{g}\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | $\mathrm{D}(\mathrm{cm})$ | $\bar{g}\left(\mathrm{~m} / \mathrm{s}^{2}\right)$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 0.394 | 3.7 | 10.06 |  |  |
| 2 | 0.423 | 3.1 | 10.17 |  |  |
| 3 | 0.449 | 2.8 | 10.39 | $\mathrm{D}=11.8$ | 10.51 |
| 4 | 0.468 | 2.7 | 10.62 |  |  |
| 5 | 0.492 | 2.4 | 10.83 |  |  |
| 6 | 0.548 | 1.9 | 11.04 |  |  |

Based on the modified method, we design a novel experiment instrument. In the modified experiment, $L$ is set as 15.2 cm , the cylinder diameter $D$ is 11.8 cm , and $l$ is set as 3.4 cm . The angles at the case of the lowest point and the highest point are denoted by $\theta_{1}$ and $\theta_{2}$, respectively. The parameters and measurements are put into Formula (5), and then the gravity acceleration is calculated. The same experiment is also repeated six times, and the average value of six experiments is taken as the experiment value. From Table 2, we can get that the gravity acceleration measured by the traditional method is 9.7413
$\mathrm{m} / \mathrm{s}^{2}$. There is a very little error between the experimental value and the accurate value $9.7988 \mathrm{~m} / \mathrm{s}^{2}$, and the relative error is $0.6 \%$. So we can conclude that the modified method can measure the gravity acceleration with high precision.

In the modified method, some measures are taken to reduce the error. Firstly, a thick bar is fixed at the center of the cylinder to help to precisely find the lowest point of the rotating liquid. Secondly, we adjust modified instrument to make the fixed arm horizontal, and make the fixed arm and the measuring arm of the angle measuring instrument in the meridian plane.

Table 2. Dates by modified method

| Number | Period (T/S) | $\theta_{1}\left({ }^{\circ}\right)$ |  | $\theta_{2}\left({ }^{\circ}\right)$ | $\mathrm{g} /\left(\mathrm{m} / \mathrm{s}^{2}\right)$ | error |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | $\left(\mathrm{m} / \mathrm{s}^{2}\right)$ |  |  |  |  |
| 1 | 0.462 | 63.15 | 35.20 | 9.6651 | $1.36 \%$ |  |
| 2 | 0.457 | 63.65 | 35.50 | 9.4140 | $3.92 \%$ |  |
| 3 | 0.466 | 68.80 | 45.60 | 10.009 | $2.15 \%$ | 9.7413 |
| 4 | 0.404 | 64.05 | 34.00 | 10.1092 | $3.17 \%$ |  |
| 5 | 0.467 | 67.95 | 43.95 | 9.8730 | $0.76 \%$ |  |
| 6 | 0.469 | 65.40 | 39.00 | 9.3773 | $4.30 \%$ |  |

## 4. CONCLUSION

A modified rotating liquid method is presented to accurately measure the gravity acceleration. The modified theoretical formula is deduced. An angle measurement instrument with an accuracy of 0.05 degree is introduced in the modified instrument. The angle measuring instrument converts the measurement of the height difference of the rotating liquid to the measurement of the angles of the highest point and the lowest point by laser. Then the gravity acceleration can be calculated easily. From the experiment data, it can be concluded that the modified method has more little error and makes the experiment easier and more convenient.

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