

Field Guide to

Light–Matter Interaction

Galina Nemova

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Introduction to the Series

2022 is a landmark year for the SPIE *Field Guide* series. It is our 19th year of publishing *Field Guides*, which now includes more than 50 volumes, and we expect another four titles to publish this year. The series was conceived, created, and shaped by Professor John Greivenkamp from the University of Arizona. John came up with the idea of a 100-page handy reference guide for scientists and engineers. He wanted these books to be the type of reference that professionals would keep in their briefcases, on their lab bench, or even on their bedside table. The format of the series is unique: spiral-bound in a 5" by 8" format, the book lies flat on any page while you refer to it.

John was the author of the first volume, the seminal *Field Guide to Geometric Optics*. This book has been an astounding success, with nearly 8000 copies sold and more than 72,000 downloads from the SPIE Digital Library. It continues to be one of the strongest selling titles in the SPIE catalog, and it is the all-time best-selling book from SPIE Press. The subsequent several *Field Guides* were in key optical areas such as atmospheric optics, adaptive optics, lithography, and spectroscopy. As time went on, the series explored more specialized areas such as optomechanics, interferometry, and colorimetry. In 2019, John created a sub-series, the *Field Guide to Physics*, with volumes on solid state physics, quantum mechanics, and optoelectronics and photonics, and a fourth volume on electromagnetics to be published this year. All told, the series has generated more than \$1.5 million in print sales and nearly 1 million downloads since eBooks were made available on the SPIE Digital Library in 2011.

John's impact on the profession through the *Field Guide* series is immense. Rival publishers speak to SPIE Press with envy over this golden nugget that we have, and this is all thanks to him. John was taken from us all too early, and to honor his contribution to the profession through this series, he is commemorated in the 2022 *Field Guides*.

Introduction to the Series

We will miss John very much, but his legacy will go on for decades to come.

Vale John Greivenkamp!

J. Scott Tyo
Series Editor, SPIE Field Guides
Melbourne, Australia, March 2022

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Preface

The interaction of light and matter has been a subject of scientific research since the 5th century BC. Its investigation has resulted in the evolution from the ancient corpuscular theory to the wave theory and finally to the quantum theory. Application of the theoretical research on light–matter interaction has led to numerous scientific achievements, including lasers, optical trapping, and optical cooling, among others. Indeed, it has brought into existence the entire field of photonics.

The primary objective of *Field Guide to Light–Matter Interaction* is to provide an overview of the basic principles of light and matter interaction using classical, semiclassical, and quantum approaches. The book covers basic photonics concepts using classical electrodynamics. A vast majority of light–matter interaction problems can be treated to a high accuracy within the semiclassical theory, where atoms with quantized energy levels interact with classical electromagnetic fields. The concepts involved in these problems are all addressed. The book also considers the interaction of matter with quantized electromagnetic fields consisting of photons. This approach gives a complete account of light–matter interaction, explaining many effects (such as the photoelectric effect) that cannot be explained using classical electromagnetic fields. The book elucidates the interaction of electromagnetic waves with atoms, molecules, solids, and plasma. It also covers the main concepts of optical pressure.

Field Guide to Light–Matter Interaction can also serve as a complement to *Field Guide to Laser Cooling Methods*, published by SPIE Press in 2019.

I would like to thank SPIE Director of Publications Patrick Franzen and Field Guide Series Editor Scott Tyo for the opportunity to write a Field Guide for one of the most interesting areas of current scientific research. I also wish to thank the anonymous reviewers for their many useful suggestions and comments on the draft of this Field Guide.

Preface

Finally, I wish to thank SPIE Press Sr. Editor Dara Burrows for her help.

This book is dedicated to my mom, Albina.

Galina Nemova
March 2022

Light and Matter in Ancient Greece

The term light–matter interaction covers a wide variety of physical phenomena ranging from classical to quantum electrodynamics. It is relevant to black holes and neutron stars. To understand the topic, one needs to answer three questions: What is light? What is matter? What is the interacting agent providing the interaction?

The first theories of light–matter interaction were developed by pre-Socratic philosophers who were active during the 5th and 6th centuries BCE in Greece.

In c. 430 BCE, **Empedocles** (c. 490–430 BCE) proposed that the world is composed of four elements, or “roots”—fire (shining Zeus), air (life-bringing Hera), earth (Aidoneus), and water (Nestis). Empedocles taught that these immutable elements are both spiritual and physical; they produce the diversity and changes in the world by the influence of two forces, repulsion (strife) and attraction (love). He did not ascribe this emotional principle to any personified existence.

Speculating about light, Empedocles believed that Aphrodite made the human eye out of the four elements and that she lit the fire in the eye, which shone out from the eye, making sight possible.

Empedocles’s idea concerning light was developed by **Euclid** (who flourished c. 300 BCE). Euclid assumed that the fire flux moved in straight lines, and he used the idea to explain some optical phenomena in his work *Optica*.

Leucippus (c. 480–420 BCE) proposed the theory of **atomism**, which was free from any traces of mythical and spiritual elements. He and **Democritus** (c. 460–370 BCE) thought that the universe comprises an infinite number of permanent, immutable, indivisible elements called **atoms** (from the Greek *atomon*, i.e., uncuttable, indivisible) and the empty space or **vacuum** (from the Latin adjective *vacuus* for vacant or void) through which the atoms move. In this ancient theory, atoms interact with each other mechanically. The theory of atomism is closer to modern science compared to any other theory of antiquity.

The Current Evolution of the Concept of Light

Controversies concerning the nature of light have a very long history.

In 1690, Christiaan **Huygens** published his book *Traité de la Lumière* [*Treatise on Light*], in which he proposed the wave theory of light.

In 1704, Sir Isaac **Newton** published *Opticks*, in which he presented his corpuscular theory of light. Newton's theory was predominant for more than 100 years. However, some scientists such as Leonard Euler, Mikhail Lomonosov, and Benjamin Franklin disagreed with Newton's corpuscular theory of light presented in *Opticks* and supported Huygens' wave theory of light.

In 1849–1850, Michel **Foucault** and Hippolyte **Fizeau** measured the speed of light. Their experiment showed that light travels more slowly through water than through air, a finding that could not be explained by Newton's corpuscular theory of light.

In 1865, James Clerk **Maxwell** published *A Dynamical Theory of the Electromagnetic Field*, where he demonstrated that electric and magnetic fields travel through space as waves moving at the speed of light.

In 1859, Gustav **Kirchhoff** experimentally investigated the intensity of the thermal electromagnetic radiation emitted by a blackbody. The classical theoretical treatment did not agree with his experimental results.

In 1900, Max **Planck** postulated that electromagnetic energy could be emitted only in quantized amounts (or packets) of energy (the **Planck postulate**). Thus, the quantum theory of light was born. Physicists now call these packets light **quanta** or **photons**.

Light propagation can be described with the **wave theory of light**. To understand the interaction between light and matter (on the atomic scale), the **quantum theory of light** must be used.

Properties of Electromagnetic Waves

In a general form, the electric and magnetic field vectors of electromagnetic waves propagating in the \vec{k} (**propagation vector**) direction can be expressed as

$$\vec{A}(x, y, z, t) = \vec{A}_0 e^{-i(\vec{k} \cdot \vec{r} - \omega t)}, \text{ where } \vec{A} \text{ is } \vec{E} \text{ or } \vec{B}$$

$$\nabla \cdot \vec{E} = 0 \rightarrow i\vec{k} \cdot \vec{E} = 0 \rightarrow \vec{E} \perp \vec{k}$$

$$\nabla \cdot \vec{H} = 0 \rightarrow i\vec{k} \cdot \vec{H} = 0 \rightarrow \vec{H} \perp \vec{k}$$

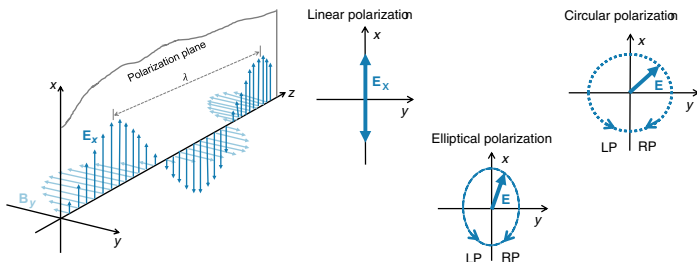
The electric field vector \vec{E} and magnetic field vector \vec{B} are perpendicular to \vec{k} as well as to each other in an infinite, nonconducting, isotropic medium. Electromagnetic waves are **transverse waves**.

A wave traveling in the z direction can be presented as two orthogonal components (polarizations):

$$\vec{E} = E_{0x} \vec{i} e^{-i(k_z z - \omega t)} + E_{0y} \vec{j} e^{-i(k_z z - \omega t + \varphi_p)}$$

where \vec{i} and \vec{j} are unit vectors in the x and y directions, respectively, and φ_p is the phase angle.

- If $\varphi_p = \pm m\pi$, where m is an integer, we have **plane** or **linearly polarized light**.



- If $E_{0x} = E_{0y}$ and $\varphi_p = \pm(2m + 1)\pi/2$, we have **circularly polarized light**, which can be right polarized (RP) or left polarized (LP); otherwise, the light is **elliptically polarized**.
- If φ_p varies in a random manner, we have **unpolarized light**.

Planck's Law for Cavity Radiation

Planck questioned the assumption that a value $\kappa_B T$ can be used as the average energy per cavity mode. He postulated that an oscillator of frequency ν can have only discrete values $n_{osc} h\nu$ of energy, where $n_{osc} = 0, 1, 2, 3, \dots$. He named the unit energy $h\nu$ a **quantum**, which cannot be further divided. In this case, the energy of each cavity mode in thermal equilibrium can be obtained using the **Boltzmann distribution** with energy $E_n = n_{osc} h\nu$. The distribution function is $f_n = C e^{-n_{osc} h\nu / \kappa_B T}$. Accounting for the normalizing condition, one can obtain the relation for C :

$$\sum_{n_{osc}=0}^{\infty} f_{n_{osc}} = \sum_{n_{osc}=0}^{\infty} C e^{-n_{osc} h\nu / \kappa_B T} = 1 \rightarrow C = 1 - e^{-h\nu / \kappa_B T}$$

The average mode energy of the oscillators is

$$\bar{E} = \frac{\sum_{n_{osc}=0}^{\infty} E_n f_n}{\sum_{n_{osc}=0}^{\infty} f_n} = \frac{h\nu}{e^{h\nu / \kappa_B T} - 1}$$

Planck replaced the $\kappa_B T$ value in the Rayleigh–Jeans formula with the average mode energy \bar{E} and obtained the relationship for the energy density per unit frequency ν for radiation within an enclosed cavity in thermal equilibrium at temperature T as follows (**Planck's formula**):

$$u(\nu) = \frac{8\pi h\nu^3}{c^3 (e^{h\nu / \kappa_B T} - 1)}$$

where h is **Planck's constant**. This formula was presented on 14 December 1900 (the “birthday” of quantum physics).

The total energy density is

$$u = \int_0^{\infty} u(\nu) d\nu = \frac{8}{15} \frac{\pi^5 \kappa_B^4}{c^3 h^3} T^4$$

In this formula, the following relation is used:

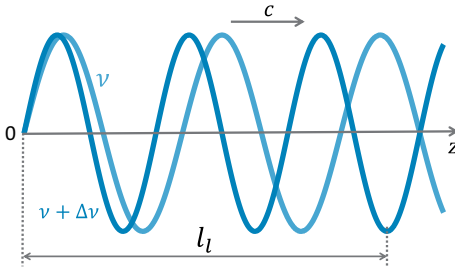
$$\int_0^{\infty} \frac{x^3 e^{-x}}{1 - e^{-x}} dx = \int_0^{\infty} x^3 e^{-x} (1 + e^{-x} + e^{-2x} + \dots) dx = \frac{\pi^4}{15}$$

Temporal and Spatial Coherence

Temporal (longitudinal) coherence describes the relative phase of two waves along their propagation direction. Consider two waves propagating along z with the frequencies ν and $(\nu + \Delta\nu)$ at the speed of light c . We assume that these two waves are in phase at $z = 0$. After the length of time $\delta_\tau = 1/\Delta\nu$ (the **coherence time**), these two waves will be out of phase. The length

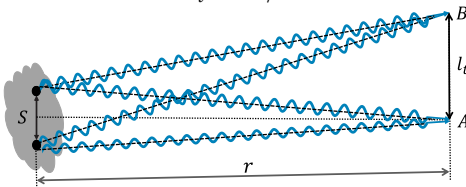
$$l_l = c\delta_\tau = c/\Delta\nu = \lambda^2/\Delta\lambda$$

is the **longitudinal coherence length**. Here, $\Delta\lambda$ is the wavelength difference between the two waves, and λ is their average wavelength. The coherence length has a significant value only when $\Delta\lambda < \lambda$, i.e., when the wavelengths of both waves are nearly identical.



Spatial (transverse or lateral) coherence describes how far apart two points radiating at frequency $\nu = c/\lambda$ can be located and still exhibit coherent properties. Consider the two light sources separated by a distance S . A is the point where these two sources manifest interference. B is the point where interference disappears. The distance between A and B is known as the **transverse coherence length**:

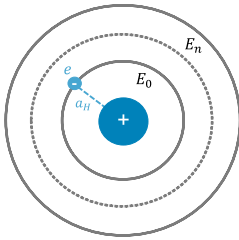
$$l_t = r\lambda/S$$



The value $V_{coh} = \pi \frac{c}{\Delta\nu} \frac{r^2 \lambda^2}{S^2}$ is the **coherence volume**.

The Bohr Theory of the Hydrogen Atom

In 1885, Johann Jakob Balmer introduced the relation $\lambda = bn^2/(n^2 - 4)$ (the **Balmer formula**), which can explain the hydrogen spectrum. Here, λ is the wavelength, $b = 3645.6$, and $n = 3, 4, 5, 6$. Unfortunately, the Balmer formula cannot be applied to any other elements, nor can it even be applied to all wavelengths emitted by hydrogen.



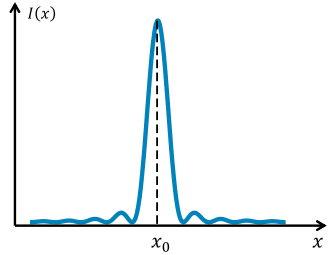
Approximately 30 years later, Niels Bohr proposed a model of the **hydrogen atom** that not only explained the Balmer formula, but also gave a new view of the atomic structure. He assumed that almost all of the mass of a hydrogen atom is concentrated in the charged nucleus. In the Bohr model, the electron, which travels in

a circular orbit about the nucleus, is attracted to the nucleus by the Coulomb force law. The magnitude of this force is $F = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$. Bohr assumed the validity of Newton's laws of motion for the orbit and calculated a centripetal acceleration of the electron as $a = \frac{v^2}{r}$. From Newton's second law of motion, $\vec{F} = m_e \vec{a}$, we have: $\frac{m_e v^2}{r} = \frac{1}{4\pi\epsilon_0} \frac{e^2}{r^2}$. The electron's kinetic and potential energies are $T_{kin} = \frac{m_e v^2}{2}$ and $V_{pot} = -\frac{e^2}{4\pi\epsilon_0 r}$, respectively. $E_{tot} = T_{kin} + V_{pot} = -\frac{e^2}{8\pi\epsilon_0 r}$ is the electron's total energy. Bohr postulated that only orbits that permit the electron's angular momentum $L = nh/2\pi$ can be used by the electron. Here, $n = 1, 2, 3, \dots$, and h is the Planck constant. The angular momentum for a circular orbit is $L = m_e v r$. In this case, the allowed energies of a hydrogen electron are $E_n = -\frac{m_e e^4}{8\epsilon_0^2 h^2} \frac{1}{n^2} = -\frac{E_0}{n^2}$ (the **Bohr formula**). E_n are **energy states** or **levels** of the atom. E_0 is the **ground state**. The allowed electron radii are $r_n = \frac{\epsilon_0 h^2}{\pi m_e e^2} n^2 = a_H n^2$, where $a_H = 0.53 \text{ \AA}$ is the **Bohr radius**. The Bohr theory provides an explanation of the spectral emission from a hydrogen atom but does not describe the structure of atoms in which multiple electrons surround the nucleus.

Wave Packet

According to the de Broglie hypothesis, a particle in free space with energy $E = \hbar\omega$ and momentum $\vec{p} = \hbar\vec{k}$ can be described by a traveling wave, which is a periodical function of $(\vec{k} \cdot \vec{r} - \omega t)$. This wave is spread over all space.

If a particle is restricted to a definite region of space, it must be described by a **wave packet**, which is a group of waves of slightly different wavelengths, with phases and amplitudes chosen such that they interfere constructively over only a small region of space. Outside of this region, these waves produce destructive interference with an amplitude that rapidly reduces to zero. The wave packet can be constructed by integrating plane waves over a small range of wavelengths. For example, in the case of light,



$$E(x) = \int_{k_0 - \Delta k}^{k_0 + \Delta k} e^{ik(x-x_0)} dk = 2 \frac{\sin[\Delta k(x-x_0)]}{(x-x_0)} e^{ik(x-x_0)}$$

where $E(x)$ is the magnitude of the electric field in the complex representation. The intensity is

$$I(x) \sim |E(x)|^2 \sim \frac{\sin^2[\Delta k(x-x_0)]}{(x-x_0)^2}$$

The destructive interference begins to be important when $|x-x_0| > 1/\Delta k$, i.e., when

$$\Delta x \Delta k \cong 1$$

Here, Δx is the width of the wave packet in x space, and Δk is the width of the wave packet in k space. The wave energy is no longer uniformly distributed throughout the space. It is localized within the Δx area. Similar results can be obtained for the time interval Δt and the angular frequency range $\Delta\omega$ needed to form such a wave packet:

$$\Delta\omega \Delta t \cong 1$$

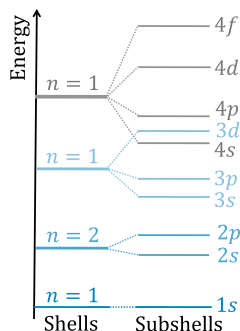
Independent-Particle Approximation

The electron–electron interaction in multi-electron atoms leads to an extremely complicated **many-body problem**. A useful approximation is to assume that each electron moves independently of all other electrons in a spherically symmetric potential created by the Coulomb field of the nucleus and the other electrons (the **independent-particle approximation**). In this approximation, an electron state can be characterized in a similar way to the electron state of a single-electron atom, i.e., with four quantum numbers such as n, l, m_l, s ; n, l, j, m_s ; or n, l, m_l, m_s .

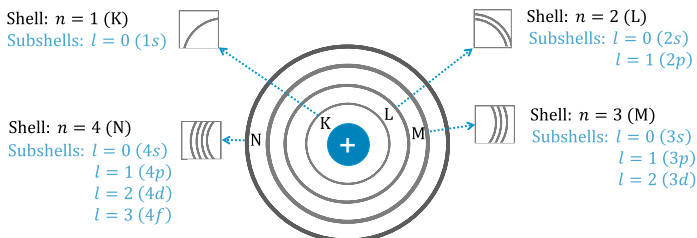
For each principal quantum number n , there are n possible values of the orbital quantum number l . For each l , there are $2l + 1$ possible values of the magnetic quantum number m_l . Each of these states has the spin magnetic quantum number $m_s = \pm 1/2$. Thus, there are $2 \sum_{l=0}^{n-1} (2l + 1) = 2n^2$ states associated with each principal quantum number n .

Electrons with the same n belong to the same **shell**. Each shell can accommodate up to $2n^2$ electrons.

Electrons with the same n and l belong to the same **subshell** ($1s$, $2s$, $2p$, etc.). Each subshell can accommodate up to $2(2l + 1)$ electrons. The subshell filling order is $1s, 2s, 2p, 3s, 3p, 4s, 3d, 4p, 5s, 4d, 5p, 6s, 4f, 5d, 6p, 7s, 5f, 6d, \dots$

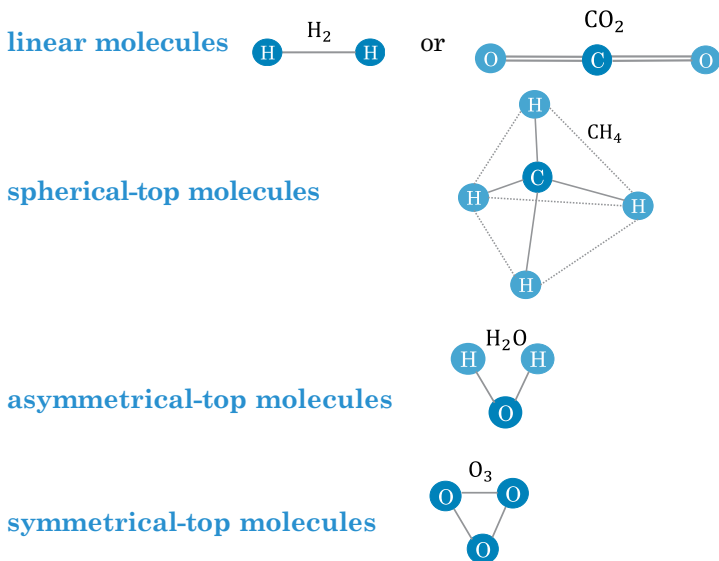


Unlike Bohr's theory, in the case of the independent-particle approximation, the energy depends on both n and l (but not on m_l or m_s).



Classification of Simple Molecules

If atoms are located within distances on the order of the atomic dimensions, short-range forces bind these atoms together to make molecules. **Simple molecules** contain a few atoms joined by **covalent bonds**, which are based on the sharing of electron pairs between atoms. Simple molecules can be divided into four basic types, which include the following:



If a molecule contains N atoms, it has $3N$ degrees of freedom, including the three translational degrees of freedom, three rotational degrees of freedom about three axes, and a number value of $(3N - 6)$ vibrational degrees of freedom.

Linear molecules have two rotational degrees of freedom about three axes, and a number value of $(3N - 5)$ vibrational degrees of freedom. As with atoms, there are only certain allowed energy levels for the electrons of a molecule. However, unlike atoms, which have only electronic energy levels, molecules also have vibrational, rotational, and electronic levels.

Rabi Frequency

The interaction of a bound electron with light can be described by the **time-dependent Schrödinger equation**,

$$i\hbar \frac{\partial \psi}{\partial t} = (H_0 + H_I)\psi$$

H_0 is the kinetic energy plus the potential energy of the electron associated with its binding to the nucleus. $H_I = -e\vec{x} \cdot \vec{E}$ is the electron–EM-field interaction energy. In a two-state system, the wavefunction for an atomic electron can be presented as $\psi(\vec{x}, t) = a_1(t)\varphi_1(\vec{x}) + a_2(t)\varphi_2(\vec{x})$, where \vec{x} is the electron–nucleus distance. We introduce $c_1(t) = a_1(t)$ and $c_2(t) = a_2(t)e^{-i\omega t}$, where ω is the field frequency. Substituting $\psi(\vec{x}, t)$ in the time-dependent Schrödinger equation, and considering that

the $e^{-i\omega t}$ terms for optical frequencies ω can be averaged to zero over a realistic time interval [the **rotating-wave approximation** (RWA)],

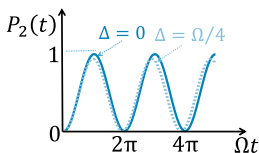
one can obtain the following relations:

$$c_1(t) = \left(\cos \frac{\Omega t}{2} + i \frac{\Delta}{\Omega} \sin \frac{\Omega t}{2} \right) e^{-i\Delta t/2}, \quad c_2(t) = i \frac{\Omega_R}{\Omega} \sin \frac{\Omega t}{2} e^{-i\Delta t/2}$$

where $c_1(0) = 1$, $c_2(0) = 0$, and $\Omega = \sqrt{\Omega_R^2 + \Delta^2}$ is the **generalized Rabi frequency**. $\Delta = \omega_0 - \omega$ is the detuning, and $\Omega_R = (e\vec{x}_{21} \cdot \hat{e})E_0\hbar^{-1}$ is the field–atom interaction energy in frequency units. It is called the **resonance Rabi frequency**. Here, $\vec{x}_{21} = \int \varphi_1^*(\vec{x})\vec{x}\varphi_2(\vec{x})d^3x$, E_0 is the amplitude of the applied electric field, and \hat{e} is the unit vector that defines the field polarization. The probabilities of finding the electron in the upper ($P_2(t) = |a_2(t)|^2$) state and lower ($P_1(t) = |a_1(t)|^2$) state are

$$P_1(t) = \frac{1}{2} \left[1 + \left(\frac{\Delta}{\Omega} \right)^2 \right] + \frac{1}{2} \left(\frac{\Omega_R}{\Omega} \right)^2 \cos \Omega t, \quad P_2(t) = \frac{1}{2} \left(\frac{\Omega_R}{\Omega} \right)^2 [1 - \cos \Omega t]$$

The generalized Rabi frequency is the frequency at which the probability oscillates between levels 1 and 2.



Light Absorption

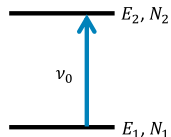
Let us consider light absorption using classical and quantum theories. In the classical absorption theory, the **absorption oscillator rate** can have the shape $S(\nu)$, which is different from the Lorentzian function. In this case, the **rate of energy absorption** by the oscillating dipole is

$$\frac{dE}{dt} = \frac{e^2}{4mc\epsilon_0} I_\nu S(\nu), \quad \text{where} \quad \int_0^\infty S(\nu) d\nu = 1$$

E is the energy of the oscillating dipole.

Consider a quantum system that consists of N similar atoms. It includes N_2 atoms at the excited energy level E_2 and $N_1 = N - N_2$ atoms at the lower energy level E_1 . Here, N_2 and N_1 are referred to as the populations of the E_2 and E_1 levels, respectively. In the quantum approach, the rate of energy absorption must be proportional to the change in population of the lower energy state, dN_1/dt , from which the absorption proceeds. Equating the rates of energy absorption in the classical and quantum approaches, one can obtain the relation

$$\frac{dE}{dt} = -h\nu_0 \frac{dN_1}{dt}$$



From this relation, we can obtain the rate of change in the population of the lower energy state caused by absorption. This rate of change has the form

$$\frac{dN_1}{dt} = -A_{21} \frac{c^2}{8\pi h \nu_0^3} N_1 I_\nu S(\nu)$$

We have considered absorption of **narrowband** radiation. In the case of **broadband** light, this relation takes the form

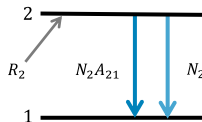
$$\frac{dN_1}{dt} = -A_{21} \frac{N_1}{8\pi h} \int_0^\infty \frac{c^2}{\nu^3} I(\nu) S(\nu) d\nu$$

Here, $I(\nu)d\nu$ is the radiation intensity in the frequency band from ν to $\nu + d\nu$.

Gain Saturation

Assume that a population inversion occurs in a medium; i.e., $\Delta N_{21} = N_2 - \frac{g_2}{g_1} N_1 > 0$. For simplicity, $g_2/g_1 = 1$. The rate equation for the upper level has the form

$$\frac{dN_2}{dt} = R_2 - N_2 \left(A_{21} + \frac{B_{21}}{c} I_\nu \right)$$



where R_2 is a pumping flux, i.e., the number of excitations per unit volume per unit time. The steady-state solution of this equation is

$$N_2 = \frac{R_2}{A_{21} + \frac{B_{21}}{c} I_\nu} = \frac{N_2^0}{1 + \frac{I_\nu}{I_\nu^{\text{sat}}}}$$

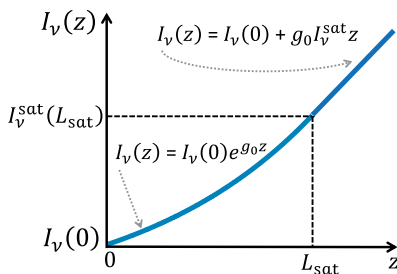
where $N_2^0 = R_2/A_{21}$ is the population of the upper level if $I_\nu = 0$. The gain $g_0 = \sigma_{21}(\nu)\Delta N_{21}^0$ is the **small-signal gain**.

If the intensity I_ν is low ($I_\nu \ll I_\nu^{\text{sat}}$), the developing beam has little effect on the population N_2 . When the beam intensity approaches the **saturation intensity** ($I_\nu \approx I_\nu^{\text{sat}}$), the stimulated emission rate becomes as large as the spontaneous emission rate. The beam intensity begins to *deplete* the population N_2 . The population difference and gain have the forms

$$\Delta N_2 = \frac{\Delta N_2^0}{1 + \frac{I_\nu}{I_\nu^{\text{sat}}}} \quad \text{and} \quad g = \frac{g_0}{1 + \frac{I_\nu}{I_\nu^{\text{sat}}}}$$

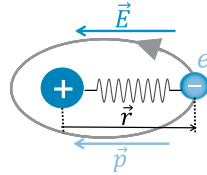
respectively. The length of the medium where saturation intensity occurs is the **saturation length** L_{sat} . A beam grows exponentially over the length L_{sat} .

If $I_\nu \gg I_\nu^{\text{sat}}$, the stimulated emission rate dominates, and the equation $\frac{dI_\nu}{dz} = gI_\nu$ can be approximated as $\frac{dI_\nu}{dz} = g_0 I_\nu^{\text{sat}}$. Its solution is $I_\nu(z) = I_\nu(0) + g_0 I_\nu^{\text{sat}} z$.



Polarization of a Dielectric Medium

Consider an **isotropic dielectric**, i.e., a material that contains no free electrons. Its positive charges are associated with the nuclei, and its negative charges are associated with the electrons bound to nuclei. The applied electric field results in a slight shift of the electron relative to its nucleus in atoms and produces an induced dipole $\vec{p} = e\vec{r}$, where e is the displaced electronic charge, and \vec{r} is the vector that locates the negative charge center relative to the positive one. The **polarization of the medium**,



$$\vec{P} = Ne\vec{r}$$

is the collective dipole moment per unit volume. Here, N is the number of dipoles per unit volume. In a simple model, electrons are coupled by springs to nuclei and behave according to Hooke's law. Applying Newton's second law to the electron in the atom, we have Newton's equation of motion:

$$m_e \frac{d^2\vec{r}}{dt^2} + m_e\gamma \frac{d\vec{r}}{dt} + k_{sp}\vec{r} = e\vec{E}$$

where k_{sp} is the spring constant, γ is a friction constant, and \vec{E} is the applied electric field. In a harmonic applied field, $\vec{E} = \vec{E}_0 e^{-i\omega t}$, the oscillator response is $\vec{r} = \vec{r}_0 e^{-i\omega t}$. The contribution to the total electric field at the position of a given dipole (due to all the other dipoles in the medium) is $\vec{P}/3\epsilon_0$. This must be added to the applied field \vec{E} . In this case, it is evident from the equation of motion that the polarization of a dielectric medium is

$$\vec{P} = \left(\frac{Ne^2}{-m_e\omega^2 - im_e\omega\gamma + k_{sp}} \right) \left(\vec{E} + \frac{\vec{P}}{3\epsilon_0} \right) \Rightarrow$$

$$\vec{P} = \left(\frac{Ne^2/m_e}{\omega_0^2 - \omega^2 - i\omega\gamma} \right) \vec{E}$$

where $\omega_0^2 = \frac{k_{sp}}{m_e} - \frac{Ne^2}{3m_e\epsilon_0}$ is the **resonance frequency**.

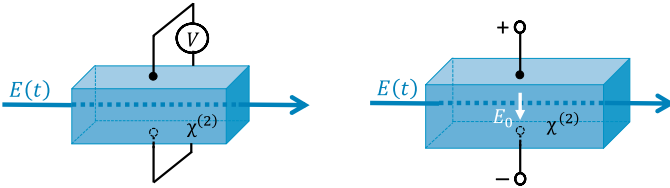
- If $\omega \ll \omega_0$, \vec{P} and \vec{E} have the same sign, and the dipoles oscillate in phase with the field.
- If $\omega \gg \omega_0$, \vec{P} and \vec{E} have a phase difference of π .
- If $\omega \approx \omega_0$, a $\pi/2$ phase shift takes place between \vec{P} and \vec{E} .

The Linear Electro-Optic Effect

Consider nonlinear media in which nonlinearities of orders higher than the second are negligible. The polarization density of these media is $P = \epsilon_0 \chi^{(1)} E + P^{NL}$, where $P^{NL} \sim \epsilon_0 \chi^{(2)} E^2$. The response of these nonlinear media to a harmonic electric field $E(t) = E_\omega \cos(\omega t)$ is

$$P^{NL} = \frac{\epsilon_0}{2} \chi^{(2)}(0; \omega, -\omega) E_\omega E_\omega + \frac{\epsilon_0}{2} \chi^{(2)}(-2\omega; \omega, \omega) E_\omega E_\omega \cos(2\omega)$$

The first term in this relation is the optical rectification or the DC electric polarization, which creates a DC voltage across a nonlinear crystal.



We apply to the crystal, besides the harmonic electric field $E(t) = E_\omega \cos(\omega t)$, a steady electric field E_0 . In this case, the **polarization density** will contain three components at the angular frequencies 0, ω , and 2ω :

$$P^{NL} = \epsilon_0 \chi^{(2)} \left[\tilde{P}_{NL}^{(0)} + \tilde{P}_{NL}^{(1)} \cos(\omega t) + \tilde{P}_{NL}^{(2)} \cos(2\omega t) \right]$$

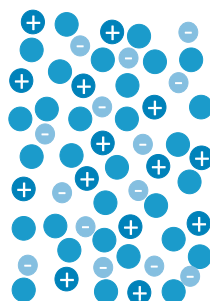
where $\tilde{P}_{NL}^{(0)} = E_0^2 + E_\omega^2/2$, $\tilde{P}_{NL}^{(1)} = 2E_0 E_\omega$, and $\tilde{P}_{NL}^{(2)} = E_\omega^2/2$.

If $E_\omega \ll E_0$, the term $\tilde{P}_{NL}^{(2)}$ may be neglected. As a result, P^{NL} will be linearized. We present $P_{NL}^{(1)} = \epsilon_0 \chi^{(2)} \tilde{P}_{NL}^{(1)}$ as $P_{NL}^{(1)} = 2\epsilon_0 \chi^{(2)} E_0 E_\omega = \epsilon_0 \Delta \chi E_\omega$, where $\Delta \chi = 2\chi^{(2)} E_0$ describes the change in the susceptibility, which is proportional to the applied steady electric field E_0 . The corresponding change in the refractive index is $\Delta n = \chi^{(2)} E_0 / n = -n^3 r E_0 / 2$, where $r = -2\chi^{(2)} / n^4$ is the **Pockels coefficient**.

*The refractive index of a medium can be modified by the applied electric field. This effect, known as the **linear electro-optic effect** or the **Pockels effect**, was first described by Fredrich Carl Pockels in 1893.*

The Debye-Hückel Length

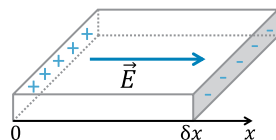
The term *plasma* was introduced by Lewi Tonks and Irving Langmuir in 1929. It comes from the Greek $\pi\lambda\alpha\sigma\mu\alpha$ and means *moldable substance*. A **plasma** is a state of matter in which all or a considerable number of atoms have lost one or several of their electrons and have become a mixture of free electrons and positive ions, which interact with one another through Coulomb forces.



The internal energy of a plasma consists of the kinetic energies of the electrons and ions, and of their Coulomb interaction energy. Here, we perturb a slab of a plasma by shifting electrons a distance x . The built-up charge is $en_e x$ per unit area, where n_e is the electron density. The magnitude of the generated electric field is

$$E = |\vec{E}| = -en_e x / \epsilon_0$$

The gain in potential energy of a charged particle after moving a distance δx through the space-charge layer is



$$W_{\text{pot}} = - \int_0^{\delta x} eE dx = \frac{e^2 n_e (\delta x)^2}{2\epsilon_0}$$

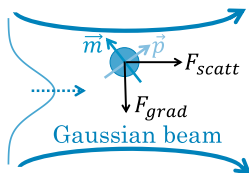
Since the mean energy associated with a degree of freedom is $\kappa_B T_e / 2$, we can estimate deviations from quasi-neutrality on a scale defined by $W_{\text{pot}} = \kappa_B T_e / 2$. This relation gives us a charge separation over the so-called **Debye-Hückel length**:

$$\lambda_D = \sqrt{\frac{\epsilon_0 \kappa_B T_e}{e^2 n_e}}$$

The Coulomb potential is screened by the medium over the distance λ_D .

Optical Force in the Rayleigh (Dipole) Approximation

If a particle radius r is much smaller than the wavelength λ of the incident light ($r \ll \lambda$), the responses of the particle to the incident light can be represented by an electric dipole \vec{p} and a magnetic dipole \vec{m} (the **Rayleigh or dipole approximation**).



According to the Mie theory, the field scattered by the particle can be calculated by summing up the contributions of all vector spherical harmonics with the Mie expansion coefficients a_ν and b_ν , where $\nu = 1, 2, 3, \dots$. If the first electric and magnetic coefficients a_1 and b_1 satisfy the inequalities $|a_1| \gg |a_m|$ and $|b_1| \gg |b_m|$, where $m \geq 2$, the polarizabilities of the particle are dominated by the first electric and magnetic components. In this case, only those terms with a_1 and b_1 need to be kept, and all other terms can be neglected. The electric and magnetic moments can be presented as $\vec{p} = \alpha_e \vec{E}$ and $\vec{m} = \alpha_m \vec{B}$, respectively. Here \vec{E} and \vec{B} are the incident electric and magnetic fields, respectively. The two polarizabilities are

$$\alpha_e = \frac{3i}{2k^3} \epsilon a_1 = \alpha_e^{(0)} \left(1 - i \frac{2}{3\epsilon} k^3 \alpha_e^{(0)} \right)^{-1}, \quad \text{where } \alpha_e^{(0)} = \epsilon r^3 \frac{\epsilon_p - \epsilon}{\epsilon_p + 2\epsilon}$$

$$\alpha_m = \frac{3i}{2k^3} \frac{b_1}{\mu} = \alpha_m^{(0)} \left(1 - i \frac{2}{3} \mu k^3 \alpha_m^{(0)} \right)^{-1}, \quad \text{where } \alpha_m^{(0)} = \frac{r^3}{\mu} \frac{\mu_p - \mu}{\mu_p + 2\mu}$$

Here μ_p and μ are the magnetic permeability, and ϵ_p and ϵ are the electric permittivity of the particle and of the surrounding medium, respectively. Considering the small particle as an electric dipole subjected to the Lorentz force in an incident electromagnetic field, one can obtain the relation

$$\vec{F} = \frac{\alpha'_e}{4} \nabla |\vec{E}|^2 + \frac{k}{2\sqrt{\epsilon}} \alpha''_e \text{Re} [\vec{E} \times \vec{B}^*] + \frac{\alpha''_e}{2} \text{Im} [(\vec{E}^* \cdot \nabla) \vec{E}]$$

where $\alpha_e = \alpha'_e + i\alpha''_e$. The *first term* is the **gradient force**, the *second term* is the **scattering force** or **radiation pressure**, and the *third term* is the **spin-curl optical force**, which is much smaller than the gradient and scattering forces.



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She was employed by Laval University (2000–2002) and Bragg Photonics Inc., Québec (2002–2004). In 2004, she joined Ecole Polytechnique de Montréal. Dr. Nemova is a Senior Member of Optica, formerly the Optical Society of America. She has edited two books and authored more than 100 papers. She is the author of *Field Guide to Laser Cooling Methods* (2019). Her research interests cover a broad range of photonics topics, including rare-earth doped materials, nanophotonics, fiber lasers and amplifiers, Raman lasers, nonlinear optics, and laser cooling of solids.