

# **Laser Beam Quality Metrics**

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# **Laser Beam Quality Metrics**

**T. Sean Ross**

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## **First rule of laser beam quality metrics:**

**Any attempt to reduce the behavior of a seven-dimensional\* object to a single number inevitably results in loss of information.**

\*three-amplitude, three-phase, and time

# Introduction to the Series

Since its inception in 1989, the Tutorial Texts (TT) series has grown to cover many diverse fields of science and engineering. The initial idea for the series was to make material presented in SPIE short courses available to those who could not attend and to provide a reference text for those who could. Thus, many of the texts in this series are generated by augmenting course notes with descriptive text that further illuminates the subject. In this way, the TT becomes an excellent stand-alone reference that finds a much wider audience than only short course attendees.

Tutorial Texts have grown in popularity and in the scope of material covered since 1989. They no longer necessarily stem from short courses; rather, they are often generated independently by experts in the field. They are popular because they provide a ready reference to those wishing to learn about emerging technologies or the latest information within their field. The topics within the series have grown from the initial areas of geometrical optics, optical detectors, and image processing to include the emerging fields of nanotechnology, biomedical optics, fiber optics, and laser technologies. Authors contributing to the TT series are instructed to provide introductory material so that those new to the field may use the book as a starting point to get a basic grasp of the material. It is hoped that some readers may develop sufficient interest to take a short course by the author or pursue further research in more advanced books to delve deeper into the subject.

The books in this series are distinguished from other technical monographs and textbooks in the way in which the material is presented. In keeping with the tutorial nature of the series, there is an emphasis on the use of graphical and illustrative material to better elucidate basic and advanced concepts. There is also heavy use of tabular reference data and numerous examples to further explain the concepts presented. The publishing time for the books is kept to a minimum so that the books will be as timely and up-to-date as possible. Furthermore, these introductory books are competitively priced compared to more traditional books on the same subject.

When a proposal for a text is received, each proposal is evaluated to determine the relevance of the proposed topic. This initial reviewing process has been very helpful to authors in identifying, early in the writing process, the need for additional material or other changes in approach that would serve to strengthen the text. Once a manuscript is completed, it is peer reviewed to ensure that chapters communicate accurately the essential ingredients of the science and technologies under discussion.

It is my goal to maintain the style and quality of books in the series and to further expand the topic areas to include new emerging fields as they become of interest to our reading audience.

*James A. Harrington  
Rutgers University*

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# Preface

This book will help the reader to thread through the subtleties of laser beam quality analysis and requirements synthesis. Chapter 1 begins with a review of basic laser properties and moves to definitions and implications of the various standard beam quality metrics such as  $M^2$ , power in the bucket, brightness, beam parameter product, and Strehl ratio. For those who are only interested in measuring Gaussian beams from commercial lasers, Chapter 1, Chapter 2 “What Your Laser Beam Analyzer Manual Didn’t Tell You,” and the first three sections of Chapter 6 “Cautionary Tales” will be sufficient. For the reader in more off-the-map areas such as unique lasers, unstable resonators, multikilowatt lasers, MOPAs, or requirements generation and development, a reading of the entire text is recommended.

The author got his start in laser metrics when assigned to align a parametric oscillator as a researcher fresh out of graduate school. After making the oscillator operational, he used a commercial beam profiler and discovered that it gave a number of either 1.3 or 7, sometimes alternating between the two in rapid succession. A perusal of the product manual added little light; everything of real interest was hidden behind the word *proprietary*. He put the commercial black box back on the shelf. Armed with a video capture card, digital camera, motion control stage, the ISO 11145:1999 standard, and LabVIEW, he built his own laser profiler, including automated  $M^2$  measurement using both the camera and knife edge. In so doing, he made just about every mistake possible and came to understand how these metrics work.

This system was used internally for several years and then retired when research needs changed. It was not until a few years later—when several hundred-million-dollar-plus laser development programs ran into trouble over the issue of laser beam quality specifications—that he realized how lacking this basic information was in the directed-energy community. It turned out to be all too easy to purchase a laser system that met specification but would not accomplish the intended task. His first beam quality publication, “Appropriate measures and consistent standard for high energy laser beam quality” was published in the Summer 2006 edition of the *Journal of Directed Energy* and won several awards. Other papers expanded the body of practical beam quality literature and were developed into a laser beam quality course that has been a regular feature of several Directed Energy Professional Society (DEPS) conferences and has been taught at SPIE’s Defense, Security, and Sensing Symposia. This text is an outgrowth of these short courses.

*T. Sean Ross  
March 2013*

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# Abbreviations, Symbols, and Notation

0	index or subscript for reference beam or fundamental mode
2D	two dimensional
3D	three dimensional
$a$	constant, aperture radius
$A$	area
$B$	brightness
BPP	beam parameter product
BQ	beam quality
$c$	speed of light in a vacuum, $\sim 2.99792 \times 10^8$ m/sec
CCD	charge-coupled device
CID	charge-injection device
$C_n$	constant with index $n$
COIL	chemical oxygen iodine laser
CW	continuous wave
$d$	differential operator. Infinitesimal change in the symbol that follows
$D$	second-moment diameter of the beam waist
DL	diffraction limited, meaning that the beam is ideal in some sense
$e$	transcendental number, 2.718281828. . . , the base of natural logarithms
$E$	energy
$\vec{E}, \mathbf{E}$	electric field
erf	error function
$f$	frequency, focal length
$F$	fluence, energy per area
$ff$	subscript for far field
FWHM	full width at half maximum
G	metric prefix meaning one billion
GHz	Gigahertz, unit of bandwidth or frequency
GW	gigawatt
HBQ	horizontal beam quality (horizontal definition of power in the bucket)

HeNe	helium–neon
$H_n[x]$	$n^{\text{th}}$ Hermite polynomial
HPIB	horizontal power in the bucket
HWHM	half width at half maximum
$\text{HW}1/e^2\text{M}$	half width at $1/e^2$ (13.5%) maximum
Hz	hertz, unit of frequency
$i$	subscript for Laguerre–Gaussian mode index
$I$	irradiance, power per area
$I_{nm}$	irradiance due to the $(n,m)^{\text{th}}$ mode
ISO	International Standards Organization
$i, j, k$	integer indices
$\hat{i}, \hat{j}, \hat{k}$	unit vectors
$j$	the imaginary number $\sqrt{-1}$
J	Joule, unit of energy
$J_n$	$n^{\text{th}}$ ordinary Bessel function
k	metric prefix meaning 1000
$k$	wavenumber $= 2\pi/\lambda$
$k_i, k[x]$	knife-edge measurement at the $i^{\text{th}}$ position, at the $x$ position
$k_{mp}$	wavenumber of $(m,p)^{\text{th}}$ mode
$K_n[x]$	$n^{\text{th}}$ modified Bessel function
kW	kilowatt, unit of power
$L$	length
$L_c$	coherence length
$L_p^m[x]$	$(p,m)^{\text{th}}$ associated Laguerre polynomial
ln	natural logarithm function
$m$	subscript for Laguerre–Gaussian mode index
mm	millimeter, unit of length
$M, M^2$	mode factor, mode factor squared. A common measure of beam quality
MOPA	master oscillator power amplifier
$N$	Fresnel number
NA	numerical aperture
ND	neutral density (filters)
Nd:YAG	neodymium-doped yttrium aluminum garnet
NEA	noise equivalent aperture
$nf$	subscript for near field
NGG	non-Gaussian Gaussian
NIST	National Institute of Standards and Technology
nm	nanometer, unit of length
nsec	nanosecond
OPA	optical parametric amplifier
OPO	optical parametric oscillator
$P$	power, energy per time

PDF	probability density function
PIB	power in the bucket, usually refers to the curve of encircled power versus radius
$q$	complex beam radius
$r$	radial spatial variable
$\bar{r}$	average radius
$R$	radius, radius of curvature
$r_0$	particular radius
rms	root mean square
$S$	Strehl ratio
SNR	signal-to-noise ratio
$t$	time
$T_c$	coherence time
$u$	electric field amplitude
$u, U$	integration variable
$v, V$	integration variable
$V$	waveguide $V$ number
$\vec{V}$	vector
VBQ	vertical beam quality (vertical definition of power in the bucket)
$V_i$	$i^{\text{th}}$ vector component
VPIB	vertical power in the bucket
$v[r]$	variable aperture measurement
$w$	beam radius
$W$	measured beam radius
$w_0, W_0$	beam radius of the fundamental mode
$w[0], W[0]$	beam waist
WFE	wavefront error
$\mathcal{W}[z]$	Lambert $W$ function, the transcendental solution of $z = \mathcal{W}e^{\mathcal{W}}$ . Implemented in <i>Mathematica</i> <sup>®</sup> as <code>ProductLog[z]</code> . Returns real values from $-1$ to infinity for arguments ranging from $-1/e$ to infinity.
$\hat{x}$	unit vector
$\bar{x}, \bar{y}$	average $x, y$
$x, y, z, X, Y, Z$	spatial variables
Yb:YLF	ytterbium-doped yttrium lithium fluoride
$Z$	zero noise level
$Z_R$	Rayleigh range
$\infty$	infinity
$\alpha$	constant
$\beta$	waveguide propagation constant
$\delta$	Dirac delta function



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$\Delta$	change in, or change of, the symbol that follows
$\Delta x$	grid spacing in the $x$ direction
$\Delta y$	grid spacing in the $y$ direction
$\Delta \lambda$	bandwidth in wavelength
$\Delta \nu$	bandwidth in frequency
$\varepsilon$	percent obscuration
$\varepsilon_0$	vacuum permittivity $8.854 \times 10^{-12}$ F/m (farad/meter)
$\theta, \Theta$	angle
$\theta_0$	divergence angle of a reference beam of fundamental mode
$\theta_{1/2}$	divergence half angle
$\theta_I$	incident angle
$\theta_r$	reflected angle
$\theta_t$	transmitted angle
$\lambda$	wavelength
$\lambda/D$	wavelength divided by near-field aperture diameter. Interpreted as the tangent of an angle. Common unit of diffraction angle for a laser beam.
$\mu\text{m}$	micron, unit of length
$\nu$	frequency
$\pi$	transcendental number, ratio of circumference of a circle to diameter, 3.14159. . .
$\sigma^2$	variance
$\sigma_n$	rms amplitude noise expressed as percent of peak
$\phi$	phase error
$\Omega$	solid angle