

Advances in Active and Nonlinear Metamaterials

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ABSTRACT

Metamaterial research is an extremely important global activity that promises to change our lives in many different ways. These include making objects invisible and the dramatic impact of metamaterials upon the energy and medical sectors of society. Behind all of the applications, however, lies the business of creating metamaterials that are not going to be crippled by the kind of loss that is naturally heralded by use of resonant responses in their construction. Under the general heading of active and tunable metamaterials, an elegant route to the inclusion of nonlinearity and waveguide complexity coupled to soliton behavior suggested by forms of transformation dynamics is presented. In addition, various discussions will be framed within a magneto-optical environment that deploys externally applied magnetic field orientations. Light can then be directed to achieve energy control and be deployed for a variety of outcomes. Quite apart from the fact that the manufacture of metamaterials is attracting such a lot of global attention, the ability to control light, for example, in these materials is also immensely interesting and will lead to a new dawn of integrated circuits and computers. Recognizing the role of nonlinearity raises the possibility that dramatic manufacturing and applications are on the horizon.

Keywords: Metamaterial, nonlinearity, soliton, magneto-optic

1. INTRODUCTION

The manufacture and application of metamaterials¹ is now a dynamic and often dramatic global activity, leading to a completely new range of devices that will have application in the energy, medical, health and computer-design sectors of society. The possibility of controlling these devices through magneto-optic environments is really exciting, because the field of magneto-optics², developed over many years, points now to readily available materials in nanostructured form. These can be easily embedded into plasmonic metamaterials. Downstream sophisticated applications should be entirely accessible, therefore. This aspiration is supported by the appearance of isotropic metamaterials³, with loss-free frequency windows, that are composites containing certain types of dielectric nanospheres and specialized magneto-optic nanospheres. With the assumption of low loss, and a Drude-Lorentz model, a considerable number of studies of how to guide light through metamaterial layers has emerged. It is difficult to see, however, that other than a two-dimensional single surface metamaterial could be envisaged as an outcome based upon split-ring/wire manufacturing approaches. That conclusion stimulates a lot of interest in deploying an isotropic, self-organised, bottom-up⁴, metamaterial for controlling waveguide modes, and the use of

impedance- matched metamaterials to negotiate bends⁵ in practical, integrated optical waveguides and even the possibility of loading hollow waveguides to create the possibility of switching between left-handed and right-handed behavior. All of this development points to viable pathways to active nonlinear devices⁶⁻¹⁰, under a general heading called integrated metaphotonics, as sketched in Fig.1, with the questions of loss, and availability of suitable materials, being widely understood and under control. In fact, the quest for waveguide manufacture may well rest upon a bottom-up approach and signal that nonlinear waveguiding, using metamaterials, can now be brought onto a much broader platform than is currently being reported. Hence, it is evident that new portals are emerging, with new frontiers ranging over a wide set of opportunities supported by magneto-optic manipulation, the role of solitons¹⁰ and transformation optics^{11,12}.

This paper addresses the growing area of nonlinear wave guiding using *active* metamaterials that are suitably doped to introduce gain, without being specific about the loss-control mechanisms. It will involve spatial solitons. In addition, vortices and vortex solitons could be important, especially if they are coupled to magneto-optic control in plasmonic metamaterials. Future applications will be placed into the context of materials being used in a light-controlling-light environment, suitable for optical chips of the future. Light-packets are important as are interfaces, couplers and complex guides with a non-uniform thickness¹³. The influence exerted by manipulating planar waveguide interfaces is another fascinating outcome that will be a feature of special metamaterials, leading to spectacular linear modal control. The first steps into the nonlinear domain will invoke weak nonlinearity but will bring out some new features of soliton propagation. This follows some interesting work, on the management of solitons, using specialised metamaterial dispositions¹⁰. This will be outlined, below in order to focus upon precisely what influence metamaterials really do have upon beam formation and propagation.

Since the first observation¹⁴ of a soliton by John Scott Russell in 1834, on a canal in Edinburgh, the potential for its application was only slowly realised but there has been a considerable impetus from the optical fibre domain. The kind of soliton that is of interest in the optical frequency range is actually analogous to solitons that can be generated on deep water and these contrast rather strongly with the KdV solitons that were observed by Russell on, relatively, shallow water. For optical systems, soliton research often divides up into an emphasis upon dispersion, or diffraction. If both are considered simultaneously then the emphasis would be upon so called optical bullets, hinc guns, but if diffraction alone is taken as the competitor to nonlinear self-focussing, then it is an area of research that comes under the heading called spatial solitons. It is the latter that will be discussed in this paper with a special emphasis on how to exploit negative phase metamaterials to achieve certain types of soliton behaviour.

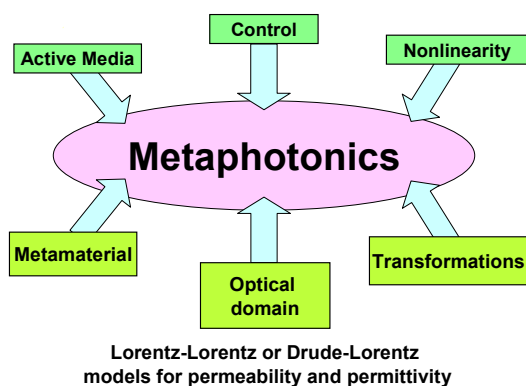


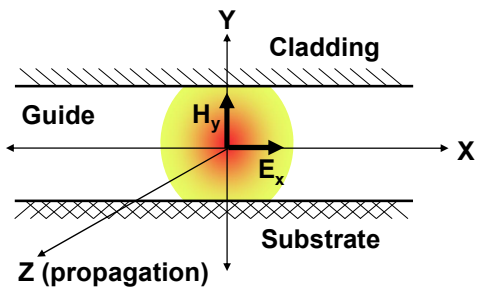
Fig.1 Defining metaphotonics

This paper devises the term metaphotonics that expresses the hope that some forms of integrated and complex geometries will be part of the mainstream metamaterial work in the near future. Under the general heading of complex structures and control, some possibilities can involve active and tunable metamaterials of the kind that was recently reviewed⁶. Metamaterials are capable of stopping light without radiation, in a frequency-selective manner. Hence, the term “the trapping of rainbows”¹³. Moving on from this, it is possible to add in some magneto-optic components to a waveguide structure of varying complexity. This heralds external control through an applied magnetic field¹⁰.

This progress towards 3-D guides, and excellent loss control is very satisfying, and truly controlled metamaterial structures are appearing on the horizon, for integrated applications. Although all of metaphotonics is the interest of this paper, a selection must be made and the choice here is to opt for the some recent development in the using nonlinear guides¹⁰. Given the extensive literature, choosing solitons to illustrate the beautiful power of nonlinearity is a natural choice and this is embraced in the next section.

2. THE SOLITON FAMILY

Throughout the last few decades there has been considerable interest in what are now known as solitons. The family of solitons is very large and, as mentioned earlier, historically, the real interest began with very shallow water waves. For optical problems, however, the hydrodynamic analogy will be nonlinear deep water waves. In all cases, some nonlinearity is being added to the system and, with the deployment of optical materials, it can be assumed that Kerr nonlinearity^{15,16}, saturated, or otherwise, is being added.



$$\frac{i\partial A}{\partial z} + \frac{\mathcal{D}}{2} \frac{\partial^2 A}{\partial x^2} + |A|^2 A + \text{additions} = 0$$

↓
 Nonlinear diffraction (κ)
 Magnetooptics (Q)

*It's a
meta
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Fig.2(a) Stable one-dimensional spatial solitons are confined to diffraction along the x-axis

Fig.2(b) This shows how the nonlinear Schrodinger equation for amplitude A is built-up to model spatial solitons. Note that \mathcal{D} is a form of diffraction management, or control. The influence through which the use of a metamaterial will have a strong influence are listed under additional effects. Magneto optic influence and nonlinear diffraction are labeled with formal parameters in the brackets.

Kerr-like nonlinearity is globally common and the metamaterials being cited here will be dominated by this type of nonlinearity behaviour. Fig. 2(a) is a sketch designed to show the propagation of an optical beam in a planar wave guide. The latter is bounded by a simple substrate and cladding. The propagation is along the z-direction, but the interesting point is that the upper and lower boundaries of the guide really do trap a beam so that diffraction is only allowed along the x-direction. The modelling task is to follow the development of its slowly varying beam amplitude, A , along the z-propagation direction. It is possible to describe what is happening to the amplitude in terms of an equation with an interesting name. The latter arises because this equation is very similar, in its basic form, to what the world has become used to calling the Schrodinger equation in the field of quantum mechanics. In photonics it is actually called the nonlinear Schrodinger equation and without any of the additions, indicated in Fig.2(b) it does reveal its structural similarity to its quantum relative. Turning specifically now to the equation in Fig. 2(b), it can be seen that it has been expressed in dimensionless form. Optical beams, like the one sketched in Fig. 2(a), experience diffraction and the essential point is that the nonlinearity opposes such diffraction, through suitable phase

adjustments and maintains the beam in its original shape. It should be noted that a parameter D is included to represent diffraction-management. This is an artificial manipulation of the diffraction through the introduction of metamaterials, and it is through this agency that metamaterials introduce a major influence. An example will be seen later on.. The equation shown in Fig. 2(b) shows the possibility of including what are called “additions” and this implies that, in addition to the basic influences, on the evolution of pulses, or beams, extra physical processes such as nonlinear diffraction can be brought into play. Nonlinear diffraction is a dominant influence when D becomes small, so its value will be cited in the figure captions. Such an additional effect depends very intimately upon the material and even more so upon the character of the metamaterial components within a guiding structure. This is really why solitons are being selected within this paper to illustrate strongly the role of metamaterials within an integrated format. In addition, it is possible to add external influences and the example selected here is going to be an externally imposed magnetic field influence within the framework of magnetooptics.in the one selected here is going to be an externally imposed magnetic field, within the framework of traditional magnetooptics.

3. DIFFRACTION-MANAGED MAGNETOOPTICS

The previous section highlighted soliton behavior and both magneto optic influence and diffraction-management was mentioned. First of all, as shown in Fig. 3, the classic role of magnetooptics revolves around the orientation of the material magnetization² relative to the propagation in, for example, a planar device. For the moment, the location of any metamaterial, either in the substrate or cladding, will not be discussed, but the orientations shown in Fig. 3 are the ones that most experimentalists will adopt. They have the classic names polar, longitudinal and transverse. In the example to be given below, it will be the transverse case that is adopted leaving polarization mixing orientations like the longitudinal case for future investigations.

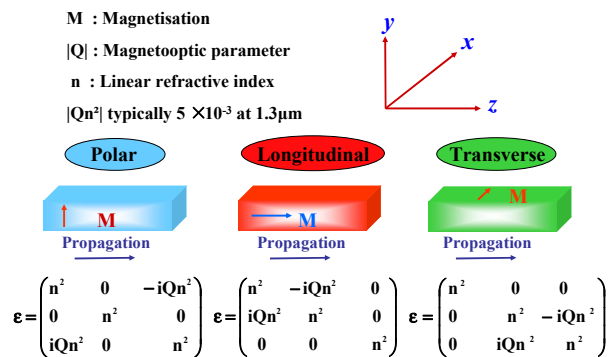


Fig.3 The classic magnetization orientations used in magnetooptics

Fig. 3 shows that the influence of an externally applied magnetic field makes its appearance as an off-diagonal element of the permittivity tensor. It is convenient to represent this influence by a parameter Q and to express the elements in terms of the zero applied magnetic field refractive index, n . Fig. 4 shows what happens to an optical beam in an integrated waveguide format that causes the beam to cross regions that are normal material, or metamaterial, in character. In Fig.4, the usual dielectric is labeled as a positive phase medium (PPM) and the kind of metamaterial being used for this example is a negative phase medium (NPM). Actually, the latter is a metamaterial in which both the relative permittivity and the relative permeability are negative (sometimes loosely referred to as double-negative). The behaviour of this periodic medium has been reported earlier¹⁰ and it must automatically be assumed that any impedance mis-match between the media has been taken care of. The external control is through an applied constant magnetic field, H_0 and this is also shown on the sketch. The system permits the beam to emerge from the diffraction-management region into an NPM region sitting on a magneto-optic substrate, for which the orientation is transverse. It is also the case that the magneto-optic parameter can be made a function of the coordinate x . In fact, in this case $Q = Q_{\max} \operatorname{sech}(x)$ in which Q_{\max} is the order of 1.8. Fig. 4 shows, dramatically, that if the external magnetic field is off, the diffraction-management forces the excitation into a very narrow beam. When this beam encounters the region where the diffraction-management ceases, it is simply radiated away. On the other hand, if the external magnetic field is on, even when the diffraction-managed narrow beam proceeds into the magneto-optic area, it is captured by a magneto-optic potential well. This illustrates beautifully how external control and internally controlled metamaterial behaviour could be used for an application. $\kappa = 1/(\epsilon(\omega)\mu(\omega)w^2)$ is a parameter measuring nonlinear diffraction, w is a beam width and $\epsilon(\omega)$ and $\mu(\omega)$ are the frequency-dependent permittivity and permeability, respectively.

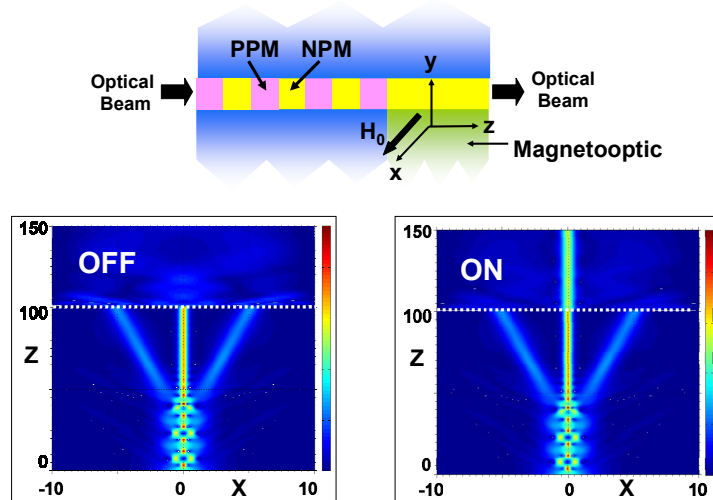


Fig.4 D=0.1: diffraction-managed optical spatial, $\kappa = 0.00168$ soliton beam created by a multilayer structure consisting of positive phase layers PPM and negative phase layers NPM. The externally applied magnetic field points along the x-axis.

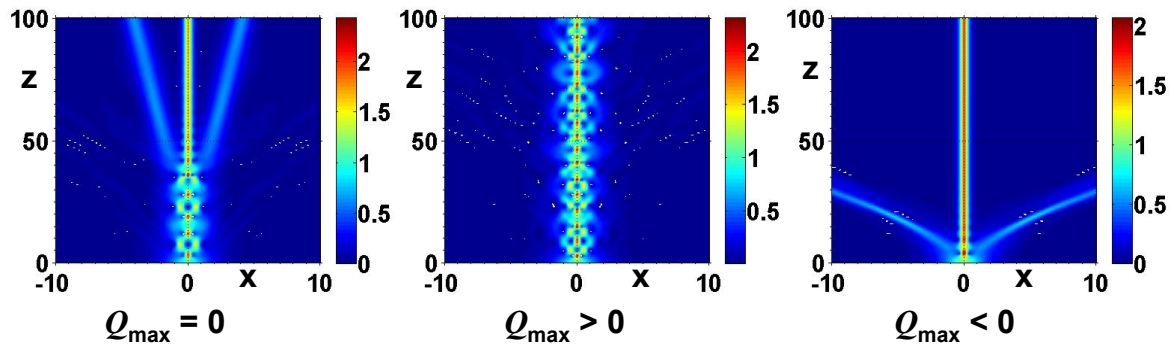


Fig. 5. $\mathcal{D} = 0.1$, $\kappa = 0.0028$ and the pictures are computed to show the influence of the external magnetic field. The normalised value of Q_{\max} is 1.8.

Fig.5 show how an applied magnetic field raise, or lower, the potential well in which a beam can sit. Diffraction-management, through the metamaterial role given to D , creates a narrow beam and the applied magnetic field either captures the beam, or causes it to radiate vigorously to a lower order. It is not discussed in detail here, but manipulating pulses¹⁷ is quite a different scenario because the additional effect added into the nonlinear Schrödinger equation is self-steepening and this can have quite a dramatic enhanced effect if a metamaterial is used. For data processing it is often necessary to enter many pulses into a system in the form of a bit pattern, and it should be expected for dense patterns with very short pulses that self-steepening would grossly distort the bit pattern. The question as to why a metamaterial would be deliberately introduced if data processing was a priority can be answered in terms of what is necessary to control guide complexity in the form of bends, for example. Given that this distortion can occur, then it is useful that an external influence can actually give a degree of control over the bit pattern. Magneto-optics gives precisely this control.

4. CONCLUSIONS

Metamaterials very easily become nonlinear and they hold out a lot of promise for life-changing applications. The global nonlinear activity is springing from a reasonably extensive groundwork in linear metamaterial guiding phenomena. It is the case, however, that even some experimental and linear design progress is still needed to move confidently to waveguiding structures based upon current metamaterial manufacturing techniques. The optimistic view is that this progress will take place and that confidence can be placed in assumptions that loss can be controlled. This may well emerge from what is known as the bottom-up approach to metamaterial creation. It is hoped that a sufficient number of references has been included here to stimulate the reader into nonlinear wave

metamaterial activity and that there is enough here to indicate that solitons may turn out to be very valuable assets to future applications. Above all, it is made clear that magneto-optics is a very valuable controlling influence. Although this paper does not address transformation optics directly, some recent progress is being made in extending the concepts of transformation optics to embrace both magneto-optics and nonlinearity.

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