Introduction to Optics

LECTURES IN OPTICS

Volume 1

George Asimellis

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COVER IMAGE:

FULL MOON RISE AT THE TEMPLE OF NEPTUNE, CAPE SOUNION, ATTICA, GREECE.

IMAGE BY THEOPHANIS MATSOPOULOS. WWW.IMDB.COM/NAME/NM7012853/

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FOREWORD

Optics is a discipline that touches nearly every aspect of our modern life. Data flows through fiber optical cables and is beamed to our displays to bring us vital information regarding our location or our favorite cat video. Optics enables us to peer deep into the cosmos to see Earth-like planets orbiting distant suns, as well as visualize the very bits of molecules that make up the essence of life. Optics enables us to peer into our bodies to find and isolate disease and allows us to implant devices into our eyes to restore and enhance vision. While there are many facets to optics, the fundamental aspects of containing and moving light are contained within geometrical optics.

George Asimellis has written *Introduction to Optics, Lectures in Optics, Volume 1*, as the first in a series of books covering the discipline of optics. Having taught optics at the undergraduate and graduate level for 20 years, I have viewed many textbooks on geometrical optics. Each of these books covers the same topics, as the material has been well understood for over 150 years. However, many of these textbooks fail to ever make it into the classroom because of their pedantic style.

Dr. Asimellis' text covers this material, but also captures the richness and spark of excitement in the applications and possibilities described above and many more. The text is a useful resource for students to gain knowledge, as well as a highly readable introduction for the casual science buff interested in learning more about optics. The writing style weaves science, history, and humor into the narrative to keep the reader engaged and provides insight into the evolution of the field. The graphics provide clear and colorful demonstrations of the concepts, while the images are visually stunning. Along the way, you will find familiar characters such as Richard Feynman and Albert Einstein, but also new players such as Chris Clavio, Carlo Bernardini, and Anish Kapoor, who create compelling artwork incorporating the fundamentals of optics. This is a rich and vibrant text that I encourage the reader to explore and enjoy.

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June 2020

PREFACE

"Geometrical Optics is either very simple or else it is very complicated."

Richard Feynman, *The Feynman Lectures on Physics*, Volume I, 27-1

Optics is fundamentally simple. At first glance, optics can be, indeed, formidably complicated. Sign conventions are difficult to memorize, the reciprocal of (meter-converted) distances involved in imaging equations are hard to rationalize, focal distances are impossible to add. These are just a few of the hurdles encountered in the ostensibly easier part of optics, that of geometrical optics. Throw into the mix the wave nature of light, the complicated integrals involved in the description of light propagation through a small aperture, or some aspects of interference and polarization, and you have the perfect recipe for confusion.

This perspective is permeated by the fact that optics instruction is fragmented, most often as part of a Physics 102 course or sometimes as part of classic electromagnetism curriculum. The presentation of optics as a whole is rare.

As a graduate student, I enrolled in two courses, one in Fourier Optics and another in Teaching Methodology. The recommended books were *Introduction to Fourier Optics* by Joseph W. Goodman and *The Feynman Lectures on Physics* by Richard Feynman. Albeit uncorrelated, these two courses changed my view of Optics forever. I appreciated how certain phenomena can be explained in a straightforward manner, for example, through a simple Fourier transform, or by the connection between quantum physics, phase diagrams, and interference.

Geometrical optics can be vastly simplified if we adhere to the Cartesian convention and the vergence method. This formulation provides a much simpler and unified tool to address imaging in geometrical optics, and, to a substantial extent, visual optics.

The philosophy that optics is simple permeates this book series. Once the reader appreciates this essential simplicity, it is a lot easier to build the foundation of fundamental knowledge, from the basics all the way to the more esoteric topics. I feel that without this understanding of the simplicity on which to build, the structure of accumulated knowledge is unsteady at best, and at worst, will crumble under its own weight.

I hope that this first volume of the *Lectures in Optics* series will be appreciated by those seeking a bottom-up textbook, fitting the needs for any college-level optics or optometry optics course.

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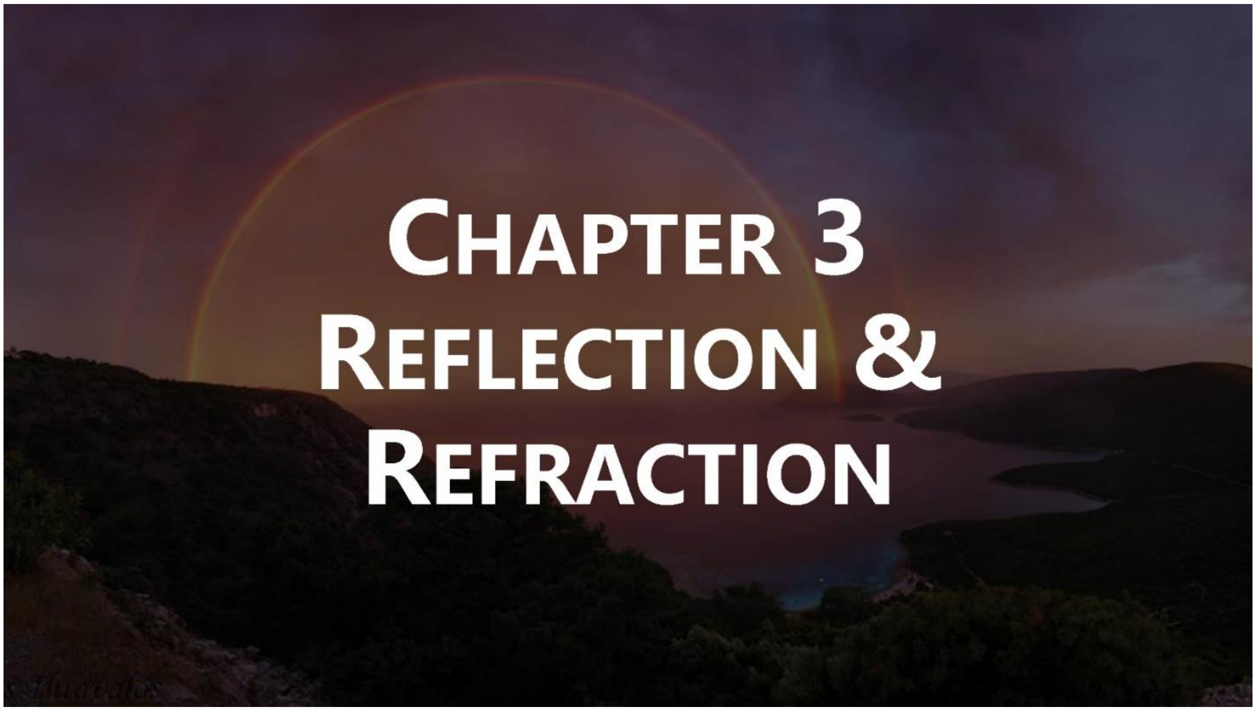
Pikeville, Kentucky
June 2020



CHAPTER 3
REFLECTION & REFRACTION

3.1 REFLECTION

Reflection is perhaps the most notable phenomenon associated with light propagation and is observed in our everyday experience. Reflection phenomena are often mentioned in art and mythology from as far back as the legend of Narcissus. Reflection phenomena are not limited to waves since they are encountered both in waves and in particles. In other words, the laws of reflection can be explained equally well by either the wave or the corpuscular theory of light.

Figure 3-1: Reflections off of a water surface. (left) White-faced ibis and (right) squacco heron (photos by Kevin Hazelgrove taken at Kalloni Saltpans, Lesvos, Greece, reprinted with permission).

3.2.1 Refraction: an Application of the Principle of Least Time

Why does light not continue its straight path in the second medium? The explanation for this behavior of light as it propagates in different materials is the change in propagation speed. Let us use the example so eloquently presented by Richard Feynman in his *Lectures in Physics*. Imagine a lifeguard on a beach where a young swimmer falls from a boat into the sea and cries for help. To save the swimmer, the lifeguard must hurry to get there soon as possible.

If the lifeguard follows a straight path, he will have to run a bit on the sand and swim a longer distance in the sea; but, because we can run faster than we can swim, this is not the quickest path. To get there as fast as possible, he should rather swim less, even if that requires a greater running distance (see the zigzag, broken-line path in Figure 3-17). This 'broken-line' path ultimately requires less total time and is the shortest 'optical' path among all others.

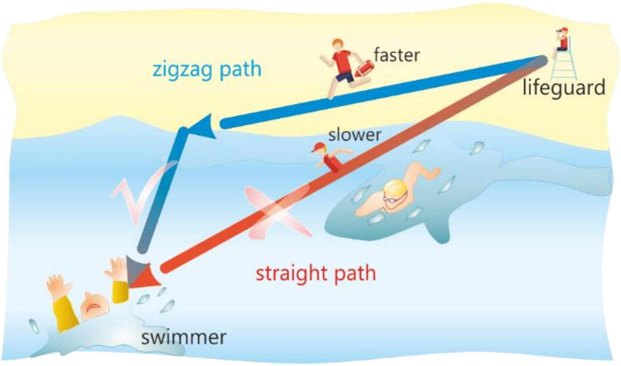


Figure 3-17: The lifeguard is in a rush to save the swimmer, so he follows a zigzag path that involves less swimming (slower) and more running on the sand (faster).

The same rule appears to govern light propagation. When light passes through an optically less dense and thus 'faster' medium such as air (where we assume propagation speed c) to an optically more dense and thus 'slower' medium such as glass or water with refractive index n, the speed of light decreases to $u = c/n$ (as defined in § 1.4). So, precisely on the dividing surface light changes direction to minimize the total time of its course, traveling less in the slow medium and, correspondingly, more in the fast medium. In other words, the laws of both reflection and refraction can be interpreted as deriving from the principle of least time.

We can arrive at the same conclusion by applying Huygens' principle (presented in § 1.2.1). Just as in the case of reflection, when the incident wave meets the dividing surface, all of the secondary waves, which now originate from the points of the dividing surface, form their own 'circle.'

3.4 REFRACTION APPLICATIONS

3.4.1 Apparent Angular Displacement

The appearance of a broken pencil when an intact pencil is submerged in water is an application of refraction. The rays that are refracted from water to the optically less dense air are bent away from the normal; the angle of refraction ϑ_t is larger than the angle of incidence ϑ_i. The pencil appears broken because we perceive the extrapolation of the rays when they reach our eyes from a virtual, apparent origin.

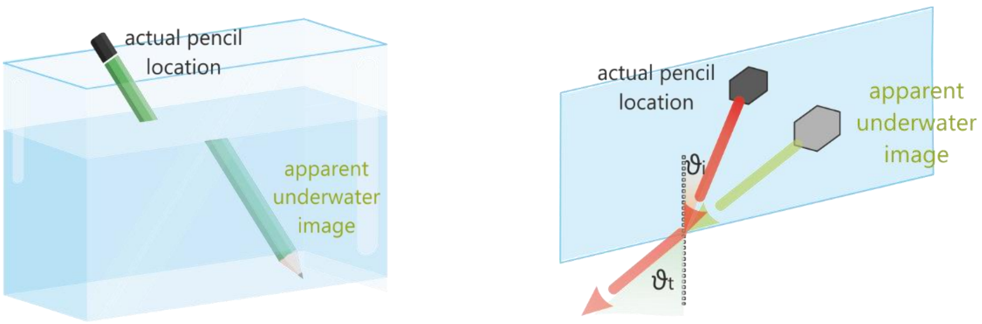


Figure 3-57: The broken pencil experiment: As rays from the pencil's underwater portion refract from water to air, the image appears displaced to the right of where the pencil is located.

The angular difference between the object and the image is the deviation angle, defined by the difference $\vartheta_t - \vartheta_i$. Assuming that the 'inner' medium is water with refractive index n_1 = 1.33, and the 'outer' medium, where observation takes place, is air with refractive index n_2 = 1.0,

$$1.33 \times \sin(\vartheta_i) = 1.0 \times \sin(\vartheta_t) \;\Rightarrow\; \vartheta_i = \sin^{-1}\left[\frac{\sin(\vartheta_t)}{1.33}\right]; \text{ thus, } \;\vartheta_t - \vartheta_i = \vartheta_t - \sin^{-1}\left[\frac{\sin(\vartheta_t)}{1.33}\right]$$

We realize that the deviation angle is dependent on the observer's position, which is determined by the refraction angle ϑ_i. For 'head on' observation, there is no deviation. This occurs when $\vartheta_i = 0°$ and thus $\vartheta_t = 0°$, which yields a deviation of 0° (this effect is the 'apparent depth,' discussed next).

We have to be looking at an oblique angle; as the observer moves away from the normal, the angle of 'observation' ϑ_t increases, as does the angular deviation between the above-water part of the object and the underwater image. This deviation can be as high as 20° if the observation angle is 60° (Figure 3-58).

3.5.1.2 Mirage Effects

Just when you may think we are done, wait—there is more. The sinking sun may appear as a doubled, reflected image not on the ocean surface, but within the atmosphere itself! The hot asphalt under the sun can produce a reflection of the sky above. These are mirage effects, which constitute a real atmospheric optical phenomenon that can be explained by stratified refraction: The air above the asphalt (or the scorching desert) reflects the sky because of strong ray bending inside the atmospheric layers due to significant thermal gradients. The name derives from the French *mirage*, which comes from *se mirer*, meaning "to be reflected."

Figure 3-71: (left) A mirage in the desert creating the illusion of water in the distance. (right) Sinking sun over a body of water. Note that the sun's second reflected image is not reflected by the water, but by layers of different temperatures in the atmosphere. (Left photo by Norbert Schmitz; right photo by Pekka Parviainen reprinted with permission.)

There are two types of mirage. The most common is the **inferior mirage**, which occurs when we observe an image of a shiny part of the sky rising from the ground. It is called inferior because the image is below the object (e.g., the sky) and is also inverted. A mirage contains at least one inverted image of an object.

An inferior mirage is created by a gradient of the atmospheric refractive index. This sounds similar to the looming sun, but there are differences. The gradient is caused by temperature variation, not hydrostatic-related density variation. At the relatively small elevation range just above hot ground, the air density (and therefore the optical density / refractive index) is prevailingly temperature-driven. When air is significantly warmer just above the desert sand, it is perhaps 10 °C cooler just 100 m higher. This exceeds the ordinary temperature gradient, which has a lapse rate of ≈ 6.5 °C/km, meaning that the air normally cools by about 6.5° C per kilometer of altitude.[25] Due to thermal expansion, this overheated air, being close to the earth, forms a localized atmospheric stratification of optically less dense layers of air.

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[25] This value applies to the standard atmosphere model described by the Committee on Extensions to the Standard Atmosphere. 1976, US Standard Atmosphere 1976 (Washington: GPO) nssdc.gsfc.nasa.gov/space/model/atmos/us_standard.html.

3.5.2.2 Halo

The legend of King Ixion (*Ἰξίων*) is associated with another prismatic optical effect, the 22° halo. Ixion was punished for being the first man in Greek mythology who was guilty of kin slaying; Jupiter (*Ζεύς*) had Mercury (*Ἑρμῆς*) bind Ixion to a winged fiery wheel. King Ixion was eternally bound to the burning solar wheel, which was spinning across the heavens.[29]

The **halo** (*ἅλως*) is a common prismatic-type atmospheric phenomenon that has some similarities to a rainbow. It manifests as circular formations when light from the sun (or the moon) is refracted by ultra-small (having diameters less than 20 µm), rod-shaped hexagonal ice crystals in the upper troposphere (5–10 km). Usually, these crystals accumulate in thin, high-level veiling clouds, such as the cirrostratus. Halos are sometimes referred to as icebows.

As light from the sun (or the moon) enters one prismatic side of a crystal, it refracts twice to exit through the opposite side with an angular deflection that has a maximum at about 22° (with a range between 21.537° for the red and 22.371° for the blue).

Figure 3-89: A 22° halo around the sun. Note that blue colors make the outer ring (left photo by Ilias Diakoumakos reprinted with permission; right photo by Selena Gallagher).

This is how the 22° halo is formed: It is composed of annular luminous rings of color that resemble the rainbow, without being a rainbow. This halo differs from the rainbow due to its formation by forward-refracted sunrays refracted by ice crystals. One difference is the order of colors: Red is on the inner rim, and blue is on the outer rim, owing to refraction geometry (Figure 3-90 right). Forward refraction also means that the halo appears along the direction of the sun and follows the solar disc. Also for this reason, the halo can be quite bright and persistent, remaining visible for hours. Unless the sun is below 22° over the horizon, the rings are completely visible, as opposed to the partially visible rings in a rainbow.

Time for practice : The principle of operation of the halo, as shown in Figure 3-90, can be demonstrated by a practical exercise of the laws of refraction. What we need to know is the refractive

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[29] Colona P. The myth of Ixion: an astronomical interpretation. Mediterranean Archaeology & Archaeometry. 2016; 16(4): 183-9.

The Human Eye as an Optical Instrument

Vision step # 1: Formation of the retinal image.

The optical system of the human eye is a combination of two positive lenses that form a real image on the retina.

4.2.2 Schematic Eyes

The optical system of the human eye is quite complex. We use certain approximations in an attempt to simplify and describe it mathematically. One of these approximations is that the optical axis is approximated by the visual axis, the line that connects the macula to the nodal points of the human eye.[37]

Schematic eyes are specific representations of the eye's basic optical elements, such as its refracting surfaces. They use specific dimensions, radii of curvature, refractive index values, etc., with a good degree of anatomic fidelity. Data for these parameters are derived from a large normative set of measurements in normal eyes.

The simplest eye model consists of just one refracting surface and is called the **reduced schematic eye**. The reduced schematic eye is a first approximation, of course, since it significantly simplifies the anatomy of the human eye. This simplified model is called Listing's reduced eye model after the German mathematician Johann Benedict Listing.

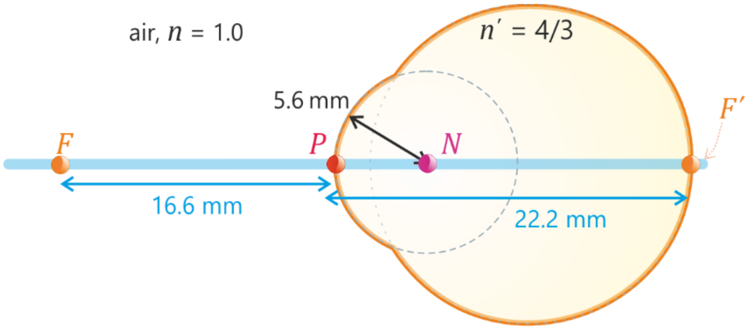

Figure 4-15: Simplified model of the human eye with just one refracting surface.

The human eye's single refracting surface corresponds to the optical interface formed by the anterior cornea and the ambient air. The eye is defined by the joint exterior surface of two spherical surfaces, which anatomically correspond to the cornea and the sclera with radii of curvature of 5.6 mm and 9.22 mm, respectively. The inner lining forms the 'screen,' the location

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[37] *Visual Optics* § 2.4.3 Axes and Reference Points in the Human Eye.

4.3 THE MAGNIFYING LENS

Our first reaction when attempting to view an object better is to bring the object closer to the eyes. The closer the object the larger it appears because the visual angle increases, as does the retinal image. How close to the eyes can we bring the object and still see it clearly?

The minimum distance that an object may be moved in order for the human eye to form a sharp retinal image is the near point, which is at ≈ 25 cm. Here the apparent angle of object observation ϑ_{25} is the largest possible. At this object distance, the retinal image is also the largest possible clear image that can be observed with the naked eye.

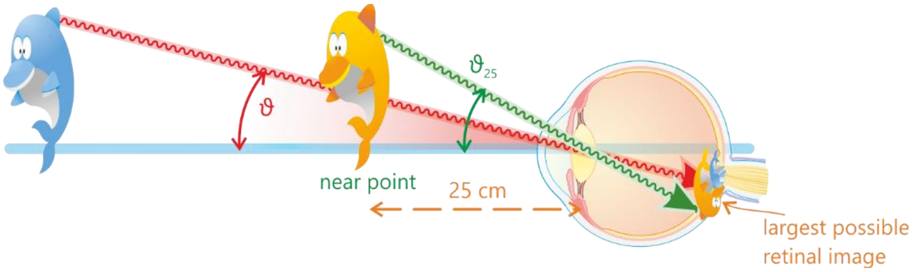

Figure 4-25: The apparent angle for an object is largest when the object is located at the near point.

We now insert a converging lens between the eye and the object; for object location x and lens focal length f, $f > |x|$.

Both the schematic solution and the analytical imaging solution (object–image vergence relationship) suggest that the virtual, erect image is formed at the location $x' < 0$ (therefore, before, or to the left of, the lens) and that $|x'| > f > |x|$. Magnification is larger than 1.0, which means that the image is larger than the object. This virtual image may now be observed with the eye.

Example ☞: The lens has a focal length $f = 10$ cm ($F = 10$ D), and the object location is $x = -6$ cm. The image is...

We implement the lens-imaging relationship: $1/x + 1/f' = 1/x'$, and we find that $x' = -15$ cm. The negative sign indicates that both the object and—more importantly—the image are located before (to the left of) the lens, 'against' the eye. Lateral magnification m is computed using the ratio of image location x' to object location x:

$m = x'/x = (-15\,\text{cm})/(-6\,\text{cm}) = +2.5$. The image is 2.5× larger than the object and is erect (+).

One of the most stunning and easily observed double stars is Albireo ($m_v = 3.0$) in Cygnus, the 'swan' constellation (β Cycni). Of its two stars, the brighter golden Albireo A ($m_v = 3.1$)[48] is in sharp contrast to the dimmer Albireo B blue-green ($m_v = 5.1$). These stars have an apparent separation of ≈ 35″. The unaided eye cannot see the two stars, but a simple 7× telescope will split this double star [Figure 5-11 (left)].

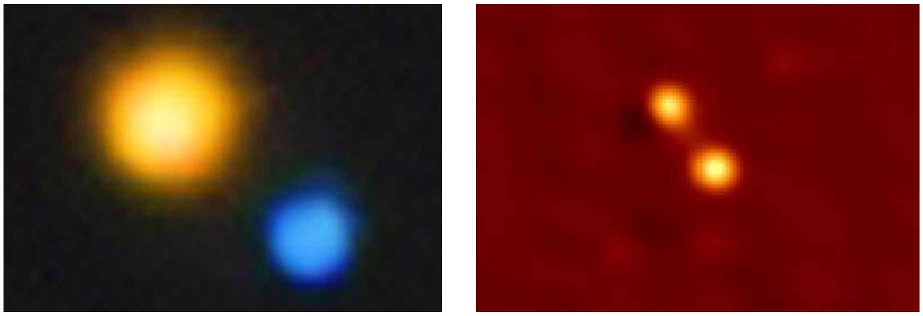

Figure 5-11: Double stars (left) Albireo viewed through a simple telescope and (right) Capella viewed through the Cambridge Optical Aperture Synthesis Telescope (COAST/MRAO). (Left image from The University of Manchester Jodrell Bank Observatory www.jodrellbank.manchester.ac.uk; right image from www.skyandtelescope.com.)

Another impressive double star is Eta Cassiopeiae, whose yellow and red stars are separated by 13 arcsec. Capella, designated α Aurigae, is the brightest star in the constellation Auriga (the 'charioteer'), the sixth brightest in the night sky, and the third brightest in the northern celestial hemisphere, with $m_v = 0.08$. It is a system of four stars grouped in two binary pairs [Figure 5-11 (right)]. The primary pair consists of two bright, type-G, giant stars, the Capella Aa and Capella Ab, which are 'suns' of about 3 times the mass of our sun. Their separation is very tight, nearly 0.05″, which is very small: 20× smaller than a 1″ separation. Capella can only be resolved by very powerful telescopes.

5.2.3.5 Useful and Maximum Magnification

The reason we magnify with the telescope is none other than to render its resolution visible to the eye (or the detector). The value of magnification that optimizes resolution is called **useful magnification**. In a manner similar to that used with a microscope, to calculate useful magnification we need the minimum angle of resolution at the eyepiece exit, which is the minimum angle of resolution of the objective times the telescope magnification:

$$\underbrace{\vartheta_{\mathrm{e\ MIN}}}_{\text{eyepiece minimum angle of resolution}} = M \times \vartheta_{\mathrm{o\ MIN}} = M \times \frac{1.22 \times \lambda}{D} \tag{5.13}$$

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[48] Albireo A has been found with advanced telescopic techniques to be two stars as well.

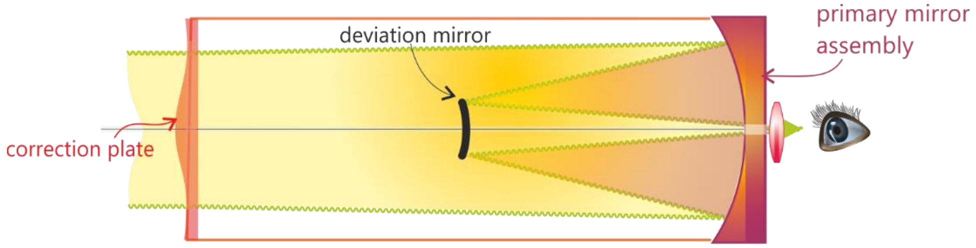

Figure 5-37: Catadioptric Schmidt-type telescope.

5.2.4.7 Modern Telescopes

5.2.4.7.1 Observing through the Atmosphere

Since their invention and until recently, telescopes had been designed as increasingly large instruments, offering a brighter and sharper view. However, once they reached a certain size, a new problem unrelated to the telescope's design was encountered: the earth's atmosphere.

Any celestial body observation from the earth ground has to pass through the atmosphere. Stars appear to twinkle, or **scintillate**. Scintillation is due to the fact that the passage of light from a pinpoint-size star though the atmosphere involves successive refractions by the various temperature and density layers/pockets, which constantly shift in a turbulent fashion (convection winds). As a starlight ray passes through these layers, it bends in unpredictable ways, resulting in slight changes in direction. What we perceive as twinkling is the starlight traveling in a broken-line path through these turbulent layers, instead of traveling a straight path. The fact that this is due to the atmosphere was proven by the lack of twinkling outside the atmosphere, as was first observed with the human eye by Walter Cunningham, an astronaut on the Apollo 7 and 8 missions. Cunningham clearly stated that stars show no obvious time variations of their visible light intensity.[53]

The atmosphere is about 100 km thick when considered as a straight path and a lot thicker if we are observing near the horizon, as scintillation may be particularly evident in low declination. Planets, on the other hand, may appear as disks (not pinpoint sources) through a telescope as they are much closer to Earth and therefore are much less affected by scintillation.

Stars, even viewed via large telescopes, appear as such twinkling points. This may be a significant problem, particularly in telescopes of large resolution and magnification. As large telescopes became sufficiently powerful, atmospheric distortion was the final obstacle. When recording a long time-average exposure, a scintillating star results in a blurry recorded image. This prevents astronomers from realizing the full potential of the more powerful telescopes.

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[53] Cunningham W, Marshall Libby L. Importance of observation that stars don't twinkle outside the earth's atmosphere. Rand Corp. 1969. Retrieved from www.rand.org/pubs/papers/P4062.html.

A lens with a small *f*/# is usually of very good quality. This is for the simple reason that such a large-diameter lens has to overcome a far larger contribution of aberrations relating to peripheral rays, such as spherical aberration, or decentered/oblique rays, such as coma and radial astigmatism.[66] Such aberrations are typically proportional to the square of the diameter. A lens with twice the diameter has to manage four times the aberrations in comparison to a lens with half the diameter, even of the same design. To manage all of such aberrations, 'fast' lenses have to be of excellent optical design and quality.

Aperture values help to create interesting artistic effects in photography. The aperture stop affects two major photographic parameters: the image brightness for a given shot time duration and the depth of field, which relates to resolving power.

Figure 6-24: Opening a lens from f/16 to f/2 results in a change in the image brightness. If the speed is fixed, then the photograph is 'dodged' (see § 6.2.3.3) For proper exposure, we need a corresponding change in speed via the shutter setting.

With smaller or larger *f*/#, the brightness doubles or halves, respectively, and this requires the shutter speed (§ 6.2.3.3) to halve or double, respectively, to compensate for the change in image brightness. This is the explanation for the terminology 'fast' and 'slow' for small and large F-numbers, respectively. A fast lens has larger light-collecting ability and is called 'fast' because it requires a shorter exposure than a slow lens.

A bright, fast lens (for instance, *f*/1.4) can collect a lot more light than a slow lens (for instance, *f*/22). Of course, these two aperture stop values can also be on the very same lens. This lens may use a 1/1000 s shot to take a fast photograph. For the same scene, if we set the lens at *f*/22, the photometer would require a speed of 1/4 s, which is a very slow shot. For this reason, the largest opening of the lens determines the classification for the lens speed.

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[66] *Geometrical Optics* § 8.3.3 Oblique/Radial Astigmatism.

George Asimellis, PhD, serves as Associate Professor of Optics and Research Director at the Kentucky College of Optometry, Pikeville, Kentucky, which he joined in 2015 as Founding Faculty. He oversees development and coordination of the Geometric Optics and Vision Science courses and development of the Laser Surgical Procedures course.

In the past, he served as head of Research at LaserVision.gr Institute, Athens, Greece, and as faculty in: the Physics Department, Aristotle University, Greece; Medical School, Democritus University, Greece; and the Electrical Engineering Department, George Mason University, Virginia.

His doctorate research involved advanced optical signal processing and pattern recognition techniques (PhD, Tufts University, Massachusetts), and optical coherence tomography (Fellowship, Harvard University, Massachusetts). He then worked on research and development of optoelectronic devices in a number in research centers in the USA. He has authored more than 75 peer-reviewed research publications, 8 scholarly books on optics and optical imaging, and a large number of presentations at international conferences and meetings.

He is on the Editorial Board of eight peer-reviewed journals, including the *Journal of Refractive Surgery*, for which he serves as Associate Editor. He received the 2017 Emerging Vision Scientist Award by the National Alliance for Eye and Vision Research (NAEVR).

His research interests include optoelectronic devices, anterior-segment (corneal and epithelial) imaging, keratoconus screening, ocular optics, and ophthalmological lasers. His recent contributions involve publications in clinical *in vivo* epithelial imaging and corneal cross-linking interventions.