Chapter 2

Digital Communication Fundamentals

2.1 Introduction

As we said at the end of Chapter 1, CDMA is applicable to digital (as opposed to analog) communication. This chapter is therefore devoted to providing an overview of basic digital communication. In its bare essence, digital communication consists of transmitting binary digits (1's and 0's) over a channel to a receiver. This process requires associating 1's and 0's with unique waveforms since the physical means of transmitting information over a wireless channel is through the conversion of information bearing waveforms into EM waves. There are also issues such as data transmission rate, noise, and modulation. These issues are addressed in this chapter beginning with the formation of the transmission waveform. The effect of noise is dealt with next, followed by that of bandwidth. The chapter concludes by looking at elementary digital modulation techniques.

2.2 Transmission and Reception in Noise

We will now introduce concepts that bear upon performance issues in digital communication. Let us assume that a `0` is transmitted as a positive pulse of amplitude A and duration T_b while a `1` is transmitted with a negative pulse of the same amplitude and duration. The quantity T_b is also referred to as the *bit interval* or the *bit period*. The data transmission rate R_b in bits per second (bps) is given by

$$R_b = \frac{1}{T_b}. \tag{2.1}$$

The waveform transmitted over the channel can therefore be represented as

$$s(t) = \sum_i a_i \, p(t - iT_b), \tag{2.2}$$

where $a_i = \pm 1$, depending on whether the ith bit is `1` or `0`, and $p(t)$ is the rectangular pulse. This waveform is shown in Fig. 2.1.

The received signal is typically regarded as the sum of the transmitted signal and a noise waveform. Usually, the noise is assumed to be zero-mean additive white Gaussian noise (AWGN). The rationale for this assumption is provided by the fact that the Gaussian distribution is widely prevalent whenever there are random phenomena. The addition of many random variables leads to nearly Gaussian distributions in many instances. In particular, if the noise process can be regarded as the sum of many independent and identically distributed random sources, then the AWGN assumption holds very well. White noise implies that the power spectrum of the noise is constant, independent of frequency. Thus the received signal $r(t)$ is written

$$r(t) = s(t) + n(t), \tag{2.3}$$

where $n(t)$ is the AWGN. The receiver must determine if, in a bit interval, the received waveform corresponds to the transmission of a 1 or a 0. In any given bit interval, the received signal is

$$r(t) = \begin{cases} A + n(t) & \text{if 1 is transmitted} \\ -A + n(t) & \text{if 0 is transmitted.} \end{cases} \tag{2.4}$$

To reduce the effect of the noise, we can average $r(t)$ over the bit interval to get

$$R = \frac{1}{T_b} \int_{\text{over } T_b} r(t)dt = a + \frac{1}{T_b} \int_{\text{over } T_b} n(t)dt = a + N, \tag{2.5}$$

where $a = \pm A$, depending on the transmitted bit, and R and N are the averages of the received signal and additive noise, respectively, over the bit interval. Since $n(t)$ is Gaussian, the random variable N is also Gaussian. Let σ^2 denote its average power. The receiver's problem now becomes one of determining, for a given value of R, whether the transmitted bit is a 1 or a 0. In the absence of noise, this is a

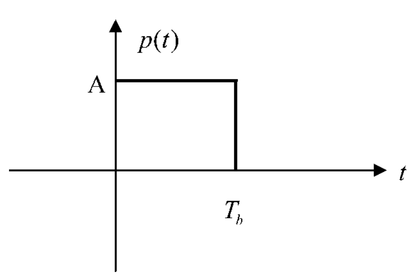

Figure 2.1 Waveform for bit transmission.

trivial problem. All the receiver has to do is decide on the basis of the sign of R; if R is positive, it knows a 1 was transmitted, otherwise a 0 was transmitted.

With noise present, we hope that averaging has reduced its contribution, then continue with the same rule:

$$\text{output bit} = 0.5 + 0.5\,\text{sign}(R), \tag{2.6}$$

where we have defined the sign function as

$$\text{sign}(x) = \begin{cases} 1 & x > 0 \\ -1 & x \leq 0. \end{cases}$$

Because of the noise term N in Eq. (2.5), it is not always the case that R is positive when a 1 is transmitted and negative when a 0 is transmitted. Thus, as a consequence of the noise, there is a finite probability of erroneous bit decisions. If 1's and 0's are equally likely to be transmitted, then the expression for the probability of error or *bit error rate* (BER) is given by[14]

$$p(e) = Q\left(\frac{A}{\sigma}\right), \tag{2.7}$$

where

$$Q(x) \triangleq \frac{1}{\sqrt{2\pi}} \int_x^\infty \exp\left(-\frac{u^2}{2}\right) du.$$

Suppose the power spectrum of the Gaussian noise is $N_0/2$ watts per Hz. Then $\sigma^2 = N_0/2T_b$ (see appendix on random signals and noise). By defining the signal-to-noise ratio (SNR) per bit as

$$\text{SNR} = \frac{A^2 T_b}{N_0/2}, \tag{2.8}$$

the expression for the BER becomes

$$p(e) = Q\left(\sqrt{\text{SNR}}\right). \tag{2.9}$$

The SNR is typically measured in decibels (dB) as

$$\text{SNR(dB)} = 10 \log(\text{SNR}). \tag{2.10}$$

A plot of the BER versus SNR is shown in Fig. 2.2.

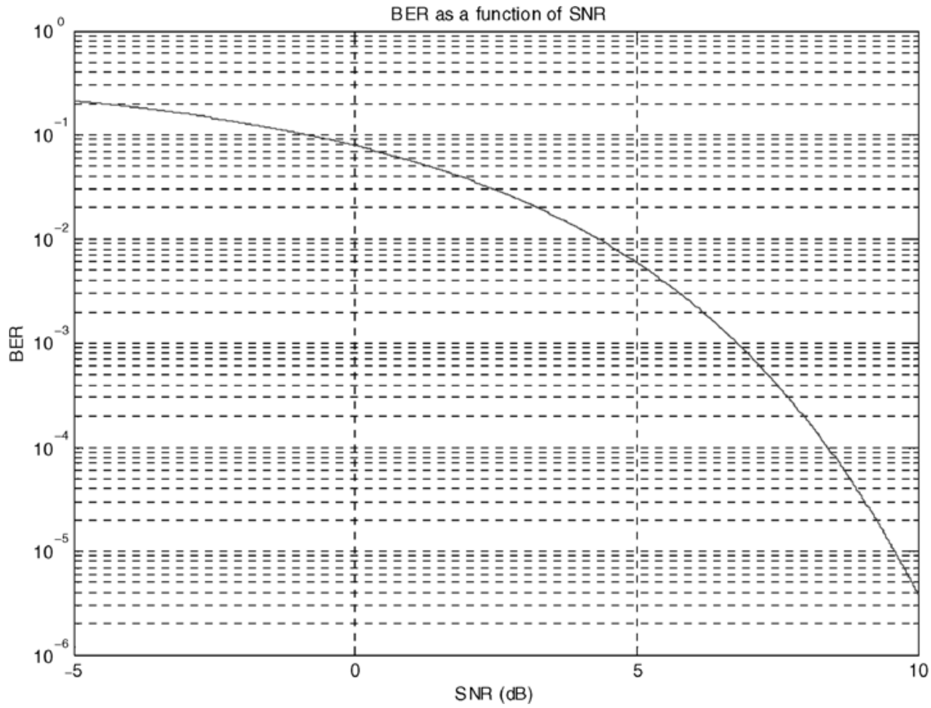

Figure 2.2 Bit error rate plot.

2.3 Effect of Finite Channel Bandwidth

It was assumed in Eq. (2.3) that the transmitted signal $s(t)$ passes through the channel and appears as is at the receiving end except for the additive noise. In practice, this does not happen. At the very least there is some loss of signal strength—this phenomenon is called *attenuation*—and delay. Suppose there is an attenuation of α and a delay of τ. Then the output of the channel is given by

$$s_0(t) = \alpha s(t - \tau), \tag{2.11}$$

where $|\alpha| \leq 1$. This expression indicates that the channel behaves as a linear time invariant (LTI) system. Since the frequency response of an LTI system is given by the ratio of the Fourier transform of the output to that of the input, the frequency response of the channel characterized by Eq. (2.11) is given by

$$H(j\omega) = \alpha \exp(-j\tau\omega). \tag{2.12}$$

Such a channel is said to provide distortionless transmission since, as can be seen from Eq. (2.11), all the information contained in $s(t)$ is preserved in $s_0(t)$. The conditions for distortionless transmission follow from Eq. (2.12) as:

1. The magnitude of the channel frequency response must be a constant, independent of frequency. This is known as the *flat frequency response condition*:

$$|H(j\omega)| = |\alpha|. \tag{2.13}$$

2. The phase of the channel frequency response obeys the linear phase condition

$$\angle H(j\omega) = -\tau\omega + m\pi, \tag{2.14}$$

   where $m = 0$ or 1 depending on whether α is positive or negative. Since the group delay $G_D(\omega)$ is defined as the negative of the derivative of the phase with respect to ω, this is also known as the *constant group delay condition* because

$$G_D(\omega) = \tau. \tag{2.15}$$

A distortionless channel is an ideal channel rarely met in practice. The channel is typically band-limited, which means that the conditions in Eqs. (2.13) to (2.15) are valid at best over a limited frequency range, and the frequency response is zero outside this range. Suppose the frequency range, in question is given by $|\omega| \leq \omega_0$. Then the channel is said to be a baseband channel with bandwidth ω_0. If, on the other hand, the frequency range is given by $\omega_1 \leq |\omega| \leq \omega_2$, the channel is said to be a bandpass channel with bandwidth $\omega_2 - \omega_1$.

If the channel is band-limited, the pulse $p(t)$ used in Eq. (2.2) should have a Fourier transform that fits within the channel bandwidth. Otherwise, the transmitted signal is distorted. The magnitude of the Fourier transform of $p(t)$ is plotted in Fig. 2.3. About 90% of the energy is in the main lobe that extends from 0 to $1/T_b$ Hz. Most of the energy, about 99%, is within $10/T_b$ Hz. Suppose the available (baseband) channel bandwidth is W Hz. Then, requiring up to 99% of the pulse energy to be within the channel bandwidth would mean

$$R_b \leq 0.1W. \tag{2.16}$$

Thus, an immediate effect of finite bandwidth is that it limits the maximum rate at which bits can be transmitted. Figure 2.4 shows the effect of finite channel bandwidth on $p(t)$ when Eq. (2.16) is violated. Here, the channel bandwidth is equal to the bit rate. Notice the distortion in the shape of the pulse.

Equation (2.16) shows that a rectangular pulse is an inefficient waveform for transmitting data over band-limited channels because it can transmit data at a rate that is only 10% of the available bandwidth. A more efficient signaling waveform, commonly used in practice, is the raised cosine waveform, which permits transmission at rates between W and $2W$ bps. The raised cosine waveform derives its name from the fact that a section of its Fourier transform consists of one-half cycle of a cosine curve raised by a constant term, as shown in Fig. 2.5. The expression

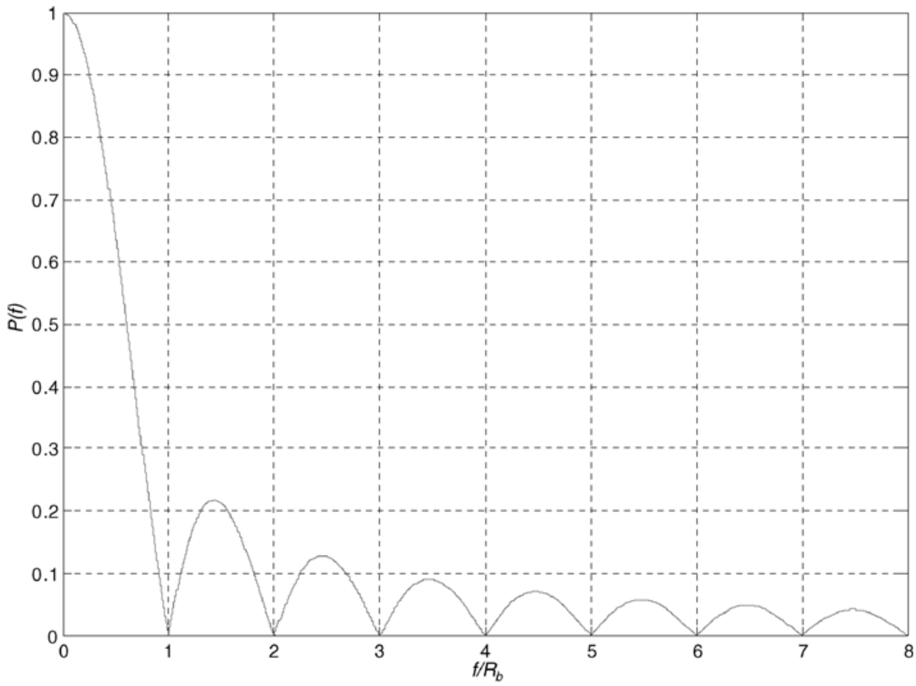

Figure 2.3 Fourier transform magnitude of rectangular pulse plotted as a function of frequency normalized by bit rate.

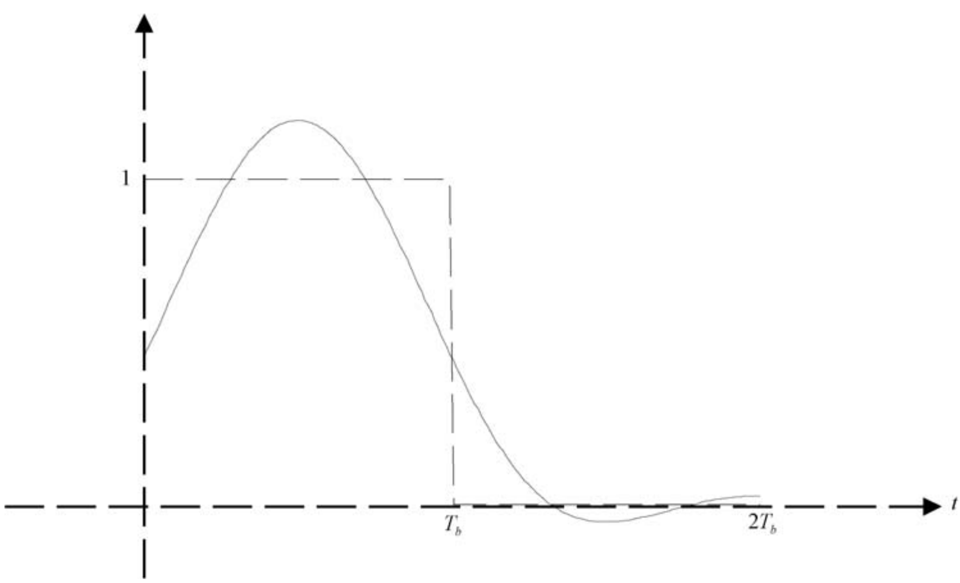

Figure 2.4 Pulse distortion due to high data rate relative to bandwidth.

Chapter 3

Code Division Multiple Access

3.1 Introduction

An alternative to frequency division and time division multiple access schemes is provided by code division multiple access (CDMA), which permits multiple users to simultaneously transmit over a channel while occupying the same frequency band. This is effected by assigning a unique code or signature to each user. As we will see, CDMA offers several advantages. Various schemes have been devised to generate the codes; this chapter will introduce the fundamentals of direct-sequence spread spectrum CDMA (DS-CDMA).

3.2 An Illustrative Example

Suppose we have two different users transmitting data simultaneously over the same wireless channel. Let us say that user 1 is transmitting data intended for destination 1 and user 2 is doing the same for destination 2. However, because of the simultaneous transmission, the received waveform at each destination is the sum of the two transmitted waveforms. Thus, each receiver has the problem of extracting just the data intended for it.

We illustrate one way of solving the problem. Let us say user 1 wants to transmit a data string d_1 given by 10110, and user 2 wants to transmit d_2 given by 11010. Suppose user 1 transmits a pulse $s_1(t)$ for a 1 and $-s_1(t)$ for a 0, where $s_1(t)$ is the rectangular pulse of duration T_b as shown in Fig. 3.1(a). On the other hand, suppose user 2 uses pulses $s_2(t)$ and $-s_2(t)$, respectively, for the same purpose, where $s_2(t)$ is shown in Fig. 3.1(b). Note that

$$\int_0^{T_b} s_i(t)s_j(t)dt = \begin{cases} T_b & i = j \\ 0 & i \neq j. \end{cases} \tag{3.1}$$

This means that the two signaling waveforms have the same energy and are orthogonal to each other.[a] The transmitted waveforms $T_1(t)$ and $T_2(t)$ for the two users

[a]The integral on the left-hand side of Eq. (3.1) is the correlation between $s_i(t)$ and $s_j(t)$. Two signals with a correlation of zero are, by definition, orthogonal to each other.

corresponding to the above data strings are shown in Fig. 3.2. The received signal at each destination is given by

$$R(t) = T_1(t) + T_2(t) \tag{3.2}$$

and is shown in Fig. 3.3.

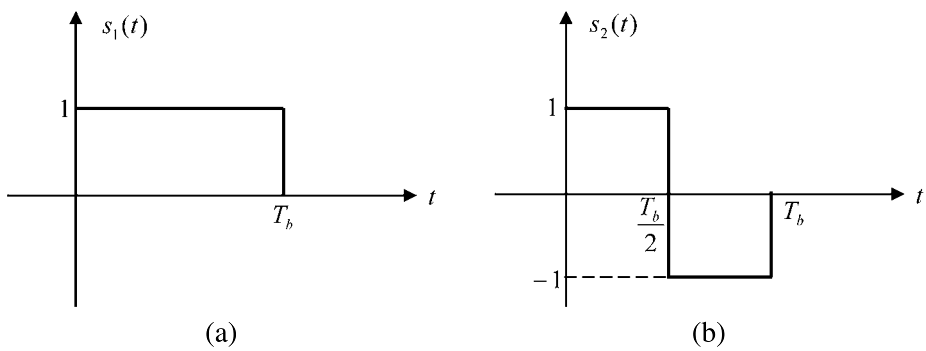

Figure 3.1 Signature waveforms for (a) user 1, and (b) user 2.

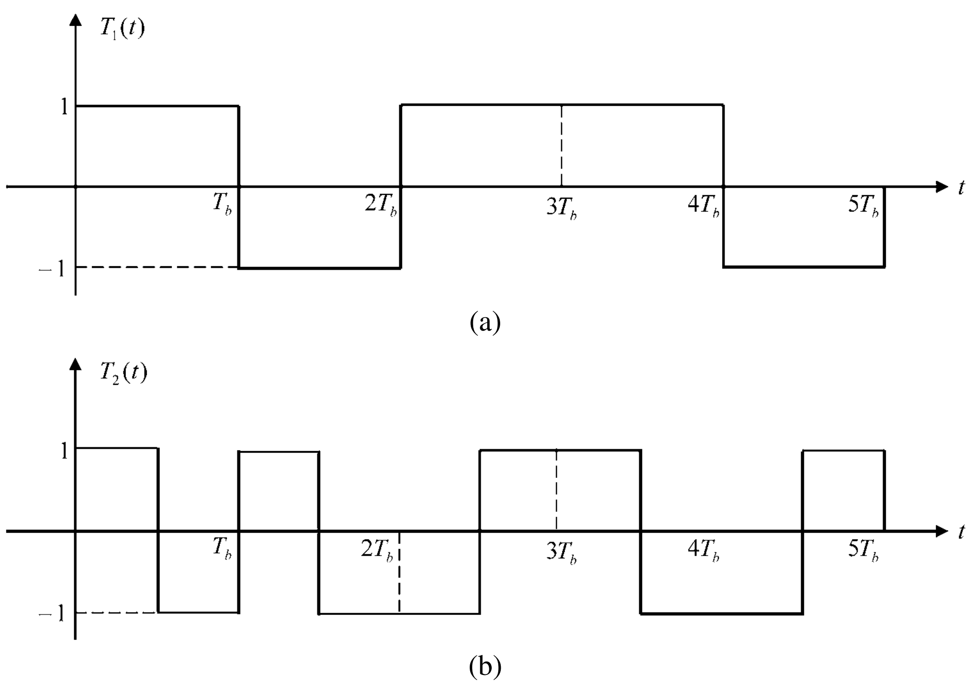

Figure 3.2 Transmitted waveforms for (a) user 1, and (b) user 2.

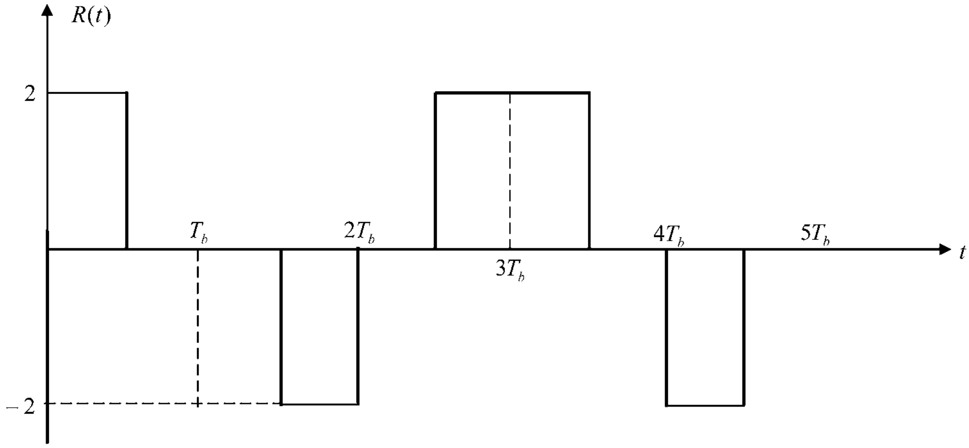

Figure 3.3 Received waveform.

Consider the first bit interval. Suppose we integrate the product of the received signal and $s_1(t)$ over this interval and call the resulting value $r_1(1)$; that is,

$$\begin{aligned} r_1(1) &= \int_0^{T_b} R(t)s_1(t)dt \\ &= \int_0^{T_b} T_1(t)s_1(t)dt + \int_0^{T_b} T_2(t)s_1(t)dt. \end{aligned} \tag{3.3}$$

For the data strings transmitted, the first integrand is $s_1^2(t)$, and the second integrand is $s_1(t)s_2(t)$. Thus, from Eq. (3.1), the first integral is T_b and the second is 0. Therefore,

$$r_1(1) = T_b. \tag{3.4}$$

Similarly, for the second bit,

$$\begin{aligned} r_1(2) &= \int_{T_b}^{2T_b} R(t)s_1(t)dt \\ &= \int_{T_b}^{2T_b} T_1(t)s_1(t)dt + \int_{T_b}^{2T_b} T_2(t)s_1(t)dt, \end{aligned} \tag{3.5}$$

the first integrand is now $-s_1^2(t)$ and the second integrand is again $s_1(t)s_2(t)$. This yields

$$r_1(2) = -T_b. \tag{3.6}$$

By repeating this process for the other bit intervals, we get

$$r_1(3) = T_b, r_1(4) = T_b, r_1(5) = -T_b. \quad (3.7)$$

By mapping T_b to 1 and $-T_b$ to 0, we find that $r_1(1)$ through $r_1(5)$ yield the string d_1. If, instead of integrating the received waveform after multiplying by $s_1(t)$ over each bit interval, we had chosen $s_2(t)$ for that purpose, the resulting values $r_2(1)$ through $r_2(5)$ would yield the string d_2. Thus, destination 1 can perform the described procedure using $s_1(t)$, and destination 2 can do the same using $s_2(t)$. In a sense, $s_1(t)$ and $s_2(t)$ are signatures of user 1 and user 2, respectively, that permit separation of the individual data strings.

The scheme can be generalized to multiple users who transmit simultaneously over the same wireless channel. Suppose there are K users sending data streams $d_1, d_2, \ldots, d_K$, where the k^{th} user's signature is $s_k(t)$. The transmitted signal is given by

$$T(t) = \sum_n [2d_1(n) - 1]s_1(t - nT_b) + \sum_n [2d_2(n) - 1]s_2(t - nT_b) + \cdots + \sum_n [2d_K(n) - 1]s_K(t - nT_b), \quad (3.8)$$

where the $2d - 1$ indicates that a data bit of 1 maps to a positive amplitude, and a data bit of 0 maps to a negative amplitude. The block diagram of the transmitter is shown in Fig. 3.4. At the receiver, the received signal is given by

$$R(t) = aT(t), \quad (3.9)$$

where a is a positive constant representing the change in amplitude experienced by the signal as it travels from transmitter to receiver.

If the signatures are pair-wise orthogonal, then the n^{th} data bit of the k^{th} user can be recovered at a receiver as

$$d_k(n) = 0.5\{1 + \text{sign}[r_k(n)]\}, \quad (3.10)$$

where

$$r_k(n) = \int_{n^{\text{th}} \text{ bit interval}} R(t)s_k(t - nT_b)dt.$$

The block diagram for the above procedure is shown in Fig. 3.5.

We have just introduced the concept of CDMA, where multiple users can transmit data simultaneously over the same channel while occupying the same bandwidth, with separation effected by unique codes or signatures associated with each user.

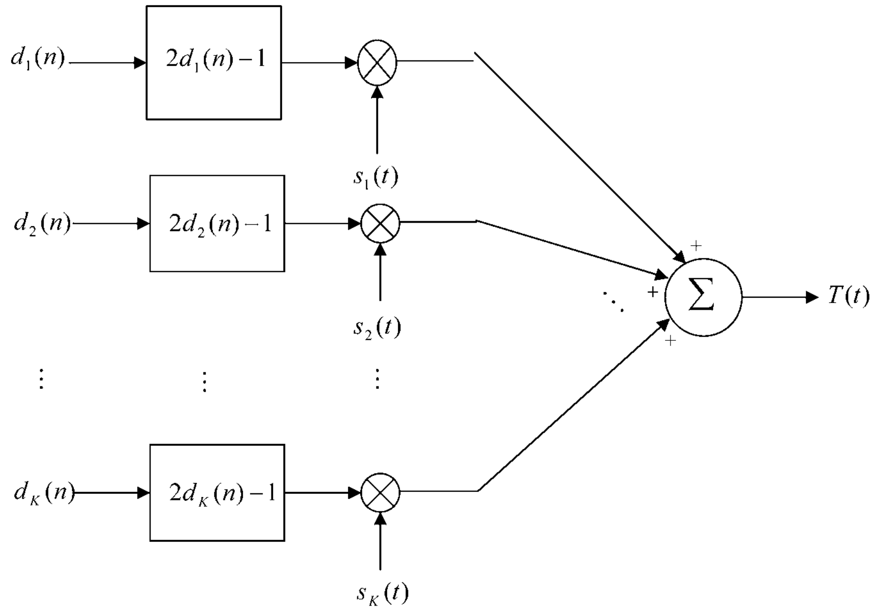

Figure 3.4 Transmitter block diagram.

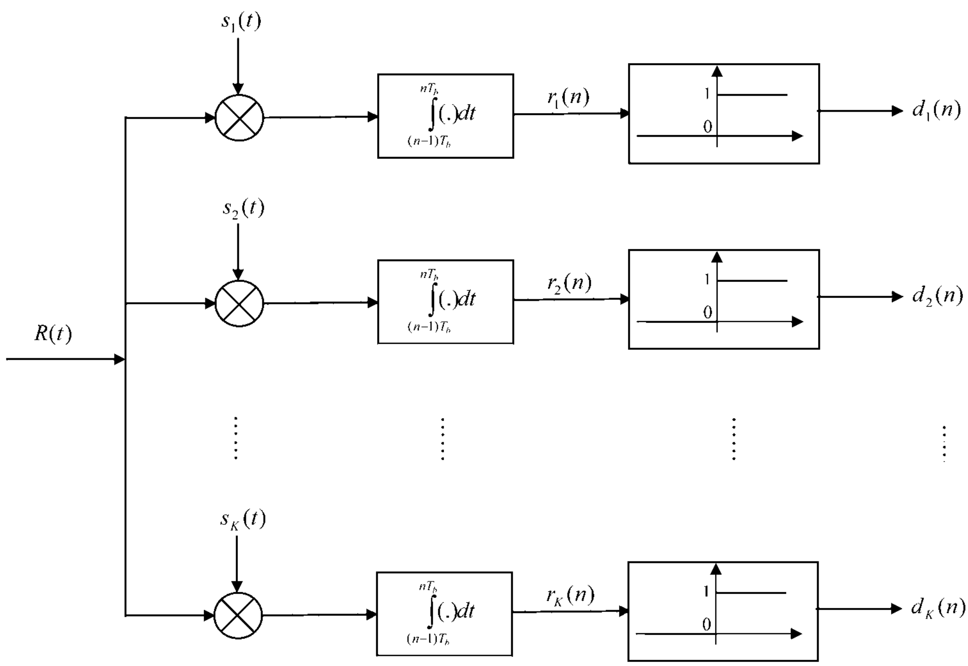

Figure 3.5 Receiver block diagram for users $1, \ldots, K$.

Generally, the received signal is contaminated by noise, and the expression in Eq. (3.10) provides only an estimate of the n^{th} data bit of the k^{th} user. The issue of noise is treated in a later chapter.