somewhat greater than 1. In this case, the fiber mode has an M^2 of 1.08, calculated by Fourier propagation methods.

1.6 Common Measures of Beam Centroid

Beam quality metrics generally require some method of determining the centroid of the beam. In the case of M^2, this is used as the basis for calculating the second-moment beam radius. In other methods, such as power in the bucket, the centroid serves as the basis for the encircled energy. Deviations in the measurement or calculation of the beam centroid affect the metric. One example of this is that, for very poor or nonradially symmetric beams, the peak value and the beam centroid are not the same, and drastically different beam quality numbers can be obtained from metrics such as power in the bucket, depending on which method is used.

1.6.1 First moment

The first moment is obtained by calculating a normalized integral of irradiance weighted by a coordinate value:

$$\bar{x} = \frac{\iint xI(x,y)\,dxdy}{\iint I(x,y)\,dxdy}; \quad \bar{y} = \frac{\iint yI(x,y)\,dxdy}{\iint I(x,y)\,dxdy};$$

$$\bar{r}^2 = \bar{x}^2 + \bar{y}^2 = \frac{\iint r^2 I(r,\theta)\,rdrd\theta}{\iint I(r,\theta)\,rdrd\theta}. \tag{1.25}$$

The first-moment integrals must be discretized for use in measurements from a digital camera:

$$x_i = x_0 + i * \Delta x; \quad y_j = y_0 + j * \Delta y;$$

$$\bar{x} = \frac{\sum_{i,j=0}^{N} x_i I(x_i, y_j)}{\sum_{i,j=0}^{N} I(x_i, y_j)}; \quad \bar{y} = \frac{\sum_{i,j=0}^{N} y_i I(x_i, y_j)}{\sum_{i,j=0}^{N} I(x_i, y_j)}. \tag{1.26}$$

1.6.2 Peak irradiance

The peak irradiance is often used in place of the centroid, especially in experimental procedures for small spots. As long as the beam is radially symmetric, this is often a very good approximation. If the wings of the beam are not concentric with the core, errors in measuring beam quality can occur.

1.6.3 Transmission maximization

The standard laboratory procedure for aligning a beam through an aperture is to place a power meter beyond the aperture and scan the aperture back and forth and

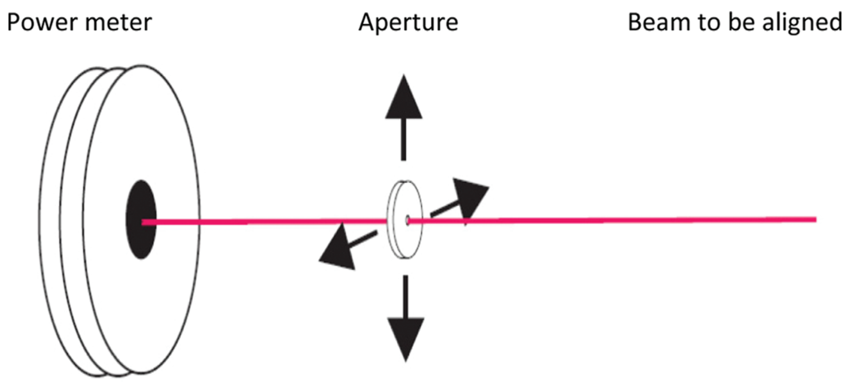


Figure 1.28 Aligning an aperture to a laser beam by maximization of transmission.

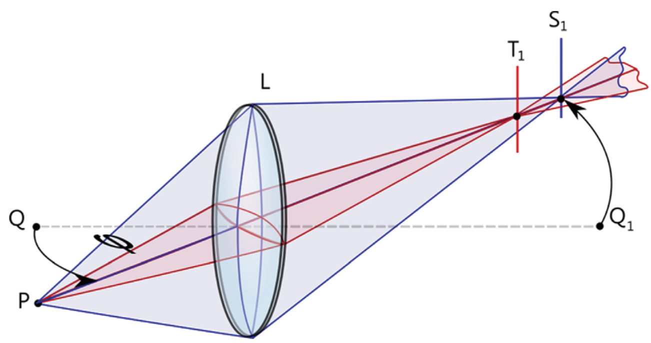
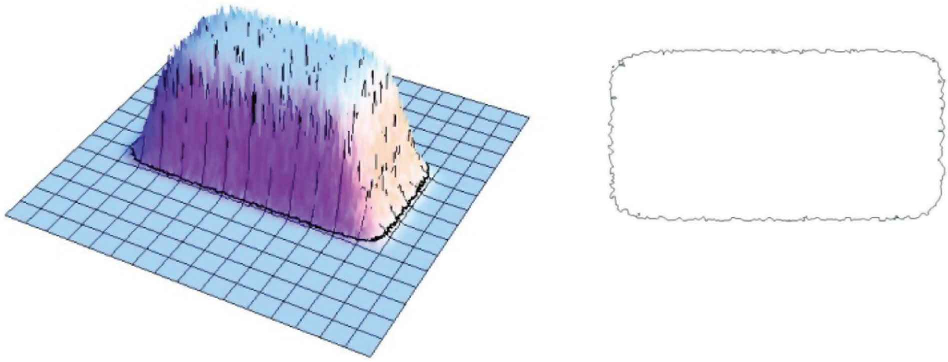


Figure 1.29 Sagittal (S_1) and tangential (T_1) foci (reproduced with permission from Sebastian Kosch/Wikimedia Commons).

up and down to maximize transmission, as shown in Fig. 1.28. If this aperture were part of a power-in-the-bucket measurement, this method would have been chosen by default, perhaps without considering its impact on the measurement of the metric. Many beams have foci in each dimension, called sagittal and tangential foci shown as T_1 and S_1 in Fig. 1.29. In this case, it might not be clear where the beam focus actually is. This can be a source of uncertainty in the metric.

1.6.4 Geometrical center/cutoff

In some cases a beam "cutoff" value is used to determine the footprint of the beam—5% of the peak value, for example. A representation of a rectangular noisy super-Gaussian irradiance profile with a 5% cutoff is shown in Fig. 1.30(a). This cutoff is used to define the geometrical shape of the beam, as in Fig. 1.30(b). The center of that footprint is sometimes used as a beam centroid. The advantage to this method is that the centroid is temporally stable even if there is a lot of noise fluctuation on the beam. In multikilowatt MOPAs, for example, amplitude noise of 400% is not unheard of. The first moment of such a beam can change significantly from shot to shot. The use of the geometric center transforms metrics such as M^2



(a) (b)

Figure 1.30 (a) Noisy rectangular super-Gaussian with 5% cutoff contour. (b) 5% cutoff contour from (a) used to calculate beam centroid.

into a closely related "times the diffraction limit" metric. As long as the cutoff footprint of the beam is within the real aperture of the beam (and this is explicitly stated), this method works for most metrics.

Two warnings need to be heeded: (1) The use of a cutoff value excludes a certain amount of energy. In the case of the beam of Fig. 1.30(a), approximately 1/2 of 1% of the total energy is outside the 5% cutoff. This might not seem significant until dealing with multikilowatt lasers. This could be 500 W in an overall 100-kW laser. It is an important principle of consistency that no laser beam quality metric excludes energy that the laser manufacturer claims in terms of total output power. The purpose of a beam quality metric is to combine with power and other output characteristics to predict the overall effect of the beam on the target. (2) Geometric footprint should not be used to calculate the area of the exit aperture for calculation of beam quality. The exit aperture needs to be the physical aperture of the system the beam is intended to pass through. A beam does not maintain its near-field geometric footprint as it passes through an optical train.

1.7 Common Measures of Beam Radius and Divergence Angle

Laser beams, in general, do not have well-defined edges. It is always a matter of concern to measure the width of an object without an obvious boundary. This is not uncommon in mechanics or physics. For example, if we were asked how to determine the diameter of the sun, our first answer might be to measure the angular extent of the bright disk that we can see and multiply by the distance from sun to earth. Yet, on further reflection, we might realize that a diameter criteria based on density, gravity, magnetic field, corona, or temperature is just as reasonable as one based on the photosphere, leading to the conclusion that the diameter of the sun is not as easy to measure as we thought. Some definitions of beam quality are intrinsically tied to a particular definition of beam radius. The second-moment radius is the method associated with M^2, and power in the bucket is tied to hard cutoff criteria. A common error in beam quality is to assume that minor changes in the beam radius method do not alter the metric. This might be true on a case-by-case basis but is not true in general. It is a mistake to assume, for example, that because half width at full maximum happens to be equal to second-moment radius in one situation, that this is the case in all situations.

Measurements of the diffraction angle of a laser beam are subject to the same innate issues as measurement of beam radius. Generally, the divergence angle θ is calculated as $\tan(\theta) = r/z$, where r is the beam radius at some far-field distance z from a waist or aperture. Every measurement or calculation of divergence angle therefore has an embedded beam radius measurement.

Modal analysis is not possible until the beam radius is determined. Depending on how the beam radius is determined, there can be many different answers to the question, "What is the mode content of that laser beam?" (see Section 1.9.8).

1.7.1 Second moment

The second moment is the weighted average of the irradiance profile with respect to a coordinate squared. The second-moment beam radius squared is twice the second-moment integral, as in Eq. (1.28). The second-moment radius of an irradiance profile is w:

$$w_x^2 = 2\frac{\iint (x - \bar{x})^2 I(x, y)\, dx dy}{\iint I(x, y)\, dx dy}; \quad w^2 = w_x^2 + w_y^2. \tag{1.27}$$

The second-moment radius is the only beam radius metric associated with M^2 (ISO, 2005). The second moment can be discretized for use with a digital camera. Its advantage is that it can be used over a variety of beam shapes. Its disadvantage is that it is fairly computationally intensive and is weighted more and more heavily the farther from the centroid one integrates. This means that stray light, camera noise, backscatter from dust, etc. can significantly alter the measurement of the second moment radius and the calculation of any beam quality metric, such as M^2, that depends on it. Experimental error in second-moment measurements will be discussed at length in Chapter 2.

1.7.2 Best fit to Gaussian

Any function can be "best fit" to a Gaussian shape by a least squares error routine, and then the width of that best fit is taken as the beam radius. It is often assumed that this method is the same as second moment. Two examples will show this to not be the case. First, a best fit to Gaussian algorithm for a Gaussian attempts to choose a parameter w to minimize the variance, as shown in Eq. (1.28) [this

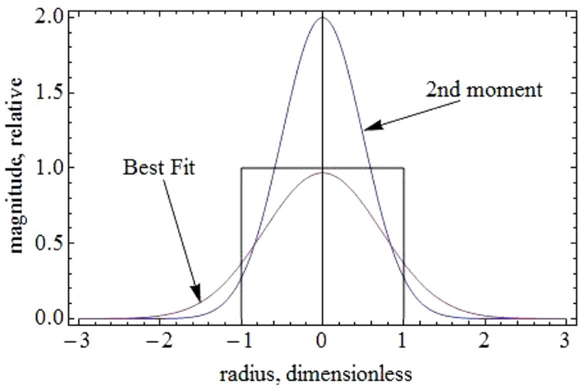


Figure 1.31 Second-moment versus best-fit radii for a cylindrical flat top.

bears little resemblance to Eq. (1.27)]:

$$\int\left(I[x,y]-\exp\left[-2\left(\frac{x^2+y^2}{w^2}\right)\right]\right)^2 dxdy. \tag{1.28}$$

A counter example shows that the minimization of Eq. (1.28) does not result in Eq. (1.27). Take the case of a circular flat-top irradiance profile of radius a. According to Eq. (1.27), converted to radial coordinates for a shape centered at the origin, the second moment is given by

$$w=\sqrt{2\frac{\iint r^2 I(r)\,r dr d\theta}{\iint I(r)\,r dr d\theta}}=\sqrt{2\frac{\int_0^a r^3 dr}{\int_0^a r dr}}=\sqrt{a^2}=a. \tag{1.29}$$

The second-moment radius of a cylindrical flat top is the radius of the flat top. Applying the best-fit routine implemented in the *Mathematica*® function, Nonlinear ModelFit returns a best-fit beam radius of $1.426a$ for a cylinder of radius a. This is shown graphically in a 2D cross section in Fig. 1.31 for a cylinder and two Gaussians of equal volumes, i.e., normalized to the same integrated power. Best fit to Gaussian and second moment are distinct measures of beam radius.

It is also worth noting some of the drawbacks to Gaussian beams. First, if the Gaussian is to have the same second-moment radius as is assumed for M^2, the Gaussian must have double the peak irradiance as a flat top. For high-energy lasers, this means that the optical damage threshold of the cavity optics is reached at a lower power level for a Gaussian than for a flat top. The next drawback to a Gaussian is that if it is to propagate as a Gaussian, it must come from an aperture two to three times larger than the flat top. The last drawback is that the Gaussian has poor energy extraction from the gain medium and generally has a lower overall power than a flat top. The apparent virtues of Gaussian beams are largely a function of unfair comparisons where the Gaussian is allowed an infinite aperture while other shapes are not. Section 6.3 on the effect of truncation on Gaussian beam propagation will discuss these issues in greater detail.

1.7.3 First null

Laser beams with hard truncation in the system generally (absent atmospheric turbulence or thermal blooming) have a well-defined central lobe with a null in the far field. Classic examples of this are the square flat top (near field) to sinc-squared (far field) and the round flat top (near field) to Airy pattern (far field) discussed in Section 1.4.2.3 The limitation of this form of measurement is that it applies primarily to the far field. Near-field beam shapes might not have obvious or stable nulls. If the application involves propagation through the atmosphere, the first null can become partially filled in or converted to a complicated shape due to laser–atmosphere interaction such as turbulence or thermal blooming. This can make metrics that depend on the first null difficult to use outside of a laboratory.

1.7.4 Hard cutoff measures

Several commonly used beam radius metrics pick an arbitrary cutoff level. These include half width at half maximum (HWHM) and half width at $1/e^2$ maximum (HW$1/e^2$M). These are easy to calculate and measure but present some drawbacks. The first drawback is that they are arbitrary and leave the beam radius open to debate. The next is that they are sometimes equal to other measures of beam radius for special cases; for example, the HW$1/e^2$M equals the second-moment radius of a Gaussian, and the half width at any cutoff equals the second moment of a circular flat top. This has spawned variations of beam quality metrics that substitute a hard cutoff measure for second moment. There is nothing wrong with this approach as long as the nonstandard metric is not called M^2. M^2, which will be discussed in Section 1.9.1 and Chapter 2, uses second-moment beam radius exclusively.

Another use of a hard cutoff measure of beam radius is the ISO two-point method, also called the knife-edge method, discussed in Section 2.8. In this case, a knife edge partially blocks the beam, and the distances between the edge location that transmits 16% and 84% of the total beam energy is taken to be the beam radius. For some cases, this method is equivalent to the second-moment method, a point that has led to another family of nontraceable variants of the M^2 metric.

1.7.5 Mode maximization

One form of Gaussian envelope theory is that of the “embedded” Gaussian, the idea being that inside an arbitrary beam shape is a zero-order Gaussian beam. One can choose the definition of beam radius that maximizes the overlap integral between a Gaussian and the beam shape. This might be an implicit choice if the beam looks *obviously* Gaussian. Section 6.2 on non-Gaussian Gaussians presents a cautionary tale illustrating this particular pitfall.